MATHCOUNTS®
COMPETITION SERIES

EST. 1983

2018-2019
school handbook

Check out this year's math problems on

pg. 11!

2018-2019 MATHCOUNTS® School Handbook

Raytheon

2019 MATHCOUNTS
National Competition Sponsor

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WestEd has recognized MATHCOUNTS as having one of the nation's most effective STEM learning programs, listing the Math Video Challenge as an Accomplished Program in STEMworks.

APPROVED FOR LISTING NASSP 2018–2019 STUDENT CONTESTS, PROGRAMS & ACTIVITIES

The National Association of Secondary School Principals has placed all three MATHCOUNTS programs on the NASSP Advisory List of National Contests and Activities for 2018-2019.

How To Use This School Handbook

If You're a New Coach

Welcome! We're so glad you're a coach this year.
Check out the **Guide for New Coaches**
starting on the next page.

If You're a Returning Coach

Welcome back! Thank you for coaching again.
Get the **2018-2019 Handbook Materials**
starting on page 8.

Guide For New Coaches

Welcome to the MATHCOUNTS® Competition Series! Thank you so much for serving as a coach this year. Your work truly does make a difference in the lives of the students you mentor. We've created this Guide for New Coaches to help you get acquainted with the Competition Series and understand your role as a coach in this program.

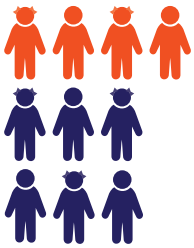
If you have questions at any point during the program year, please feel free to contact the MATHCOUNTS national office at info@mathcounts.org.

The MATHCOUNTS Competition Series in a Nutshell

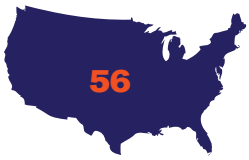
The **MATHCOUNTS Competition Series** is a national program that provides students the opportunity to compete in live, in-person math contests against and alongside their peers. Created in 1983, it is the longest-running MATHCOUNTS program and is open to all sixth-, seventh- and eighth-grade students.

HOW DOES IT WORK? The Competition Series has 4 levels of competition—school, chapter, state and national. Here's what a typical program year looks like.

Schools register in the fall and work with students during the year. Coaches administer the School Competition, usually in January. Any number of students from your school can participate in your team meetings and compete in the School Competition. MATHCOUNTS provides the School Competition to coaches in November. Many coaches use this to determine which student(s) will advance to the Chapter Competition.



Between 1 and 10 students from each school advance to the local Chapter Competition, which takes place in February. Each school can send a team of 4 students plus up to 6 individual competitors. All chapter competitors—whether they are team members or individuals—participate in the individual rounds of the competition; then just the 4 team members participate in the team round. Schools also can opt to send just a few individual competitors, rather than forming a full team. Over 500 Chapter Competitions take place across the country.



Top students from each Chapter Competition advance to their State Competition, which takes place in March. Your school's registration fees cover your students as far as they get in the Competition Series. If your students make it to one of the 56 State Competitions, no additional fees are required.

Top 4 individual competitors from each State Competition receive an all-expenses-paid trip to the National Competition, which takes place in May. These 224 students combine to form 4-person state teams, while also competing individually for the title of National Champion.

WHAT DOES THE TEST LOOK LIKE? Every MATHCOUNTS competition consists of 4 rounds—Sprint, Target, Team and Countdown Round. Altogether the rounds are designed to take about 3 hours to complete. Here's what each round looks like.

Sprint Round
40 minutes
30 problems total
no calculators used
focus on speed and accuracy

Target Round
Approx. 30 minutes
8 problems total
calculators used
focus on problem-solving and mathematical reasoning

The problems are given to students in 4 pairs. Students have 6 minutes to complete each pair.

Team Round
20 minutes
10 problems total
calculators used
focus on problem-solving and collaboration

Only the 4 students on a school's team can take this round officially.

Countdown Round
Maximum of 45 seconds per problem
no calculators used
focus on speed and accuracy

Students with highest scores on Sprint and Target Rounds compete head-to-head. This round is optional at the school, chapter and state level.

HOW DO I GET MY STUDENTS READY FOR THESE COMPETITIONS? What specifically you do to prepare your students will depend on your schedule as well as your students' schedules and needs. But in general, working through lots of different MATHCOUNTS problems and completing practice competitions is the best way to prepare to compete. Each year MATHCOUNTS provides the *School Handbook* to all coaches, plus lots of additional free resources online.

The next sections of this Guide for New Coaches will explain the layout of the *MATHCOUNTS School Handbook* and other resources, plus give you tips on structuring your team meetings and preparation schedule.

The Role of the Competition Coach

Your role as the coach is such an important one, but that doesn't mean you need to know everything, be a math expert or treat coaching like a full-time job. Every MATHCOUNTS coach has a different coaching style and you'll find the style that works best for you and your students. But in general **every good MATHCOUNTS coach must do the following**.

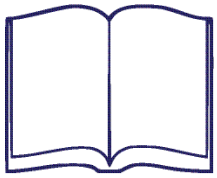
- Schedule and run an adequate number of practices for participating students.
- Help motivate and encourage students throughout the program year.
- Select the 1-10 student(s) who will represent the school at the Chapter Competition in February.
- Take students to the Chapter Competition or make arrangements with parents and volunteers to get them there.

You don't need to know how to solve every MATHCOUNTS problem to be an effective coach. In fact, many coaches have told us that they themselves improved in mathematics through coaching. Chances are, you'll learn with and alongside your students throughout the program year.

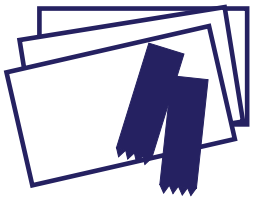
You don't need to spend your own money to be an effective coach. You can prepare your students using solely the free resources and this handbook. We give coaches numerous detailed resources and recognition materials so you can guide your Mathletes® to success even if you're new to teaching, coaching or competition math, and even if you use only the free resources MATHCOUNTS provides all competition coaches.

Making the Most of Your Resources

As the coach of a registered competition school, you already have received what we at MATHCOUNTS call the **School Competition Kit**. Your kit includes the following materials for coaches.



2018-2019 MATHCOUNTS School Handbook
The most important resource included in the School Competition Kit. Includes 250 problems.



Student Recognition Ribbons and Certificates
10 participation certificates, 1 Champion ribbon and 1 Second Place ribbon.

You'll also get access to electronic resources. The following resources are available to coaches online at **www.mathcounts.org/coaches**. This section of the MATHCOUNTS website is restricted to coaches and you already should have received an email with login instructions. *If you have not received this email, please contact us at info@mathcounts.org to make sure we have your correct email address.*

Official 2019 MATHCOUNTS School Competition
Released in November 2018
Includes all 4 test rounds and the answer key

2018 MATHCOUNTS School, Chapter + State Competitions
Released by mid-April 2018
Each level includes all 4 test rounds and the answer key

MATHCOUNTS Problem of the Week
Released each Monday
Each multi-step problem relates to a timely event

You can use the **2019 MATHCOUNTS School Competition** to choose the students who will represent your school at the Chapter Competition. Sometimes coaches already know which students will attend the Chapter Competition. If you do not need the School Competition to determine your chapter competitors, then we recommend using it as an additional practice resource for your students.

The **2018-2019 MATHCOUNTS School Handbook** will be your primary resource for the Competition Series this year. It is designed to help your students prepare for each of the 4 rounds of the test, plus build critical thinking and problem-solving skills. This section of the Guide for New Coaches will focus on how to use this resource effectively for your team.

WHAT'S IN THE HANDBOOK? There is a lot included in the *School Handbook*, and you can find a full table of contents on pg. 8 of this book, but below are the sections that you'll use the most when coaching your students.

- **Handbook Problems:** 250 math problems divided into Warm-Ups, Workouts and Stretches. These problems in-

crease in difficulty as the students progress through the book. (pg. 11)

- **Solutions to Handbook Problems:** complete step-by-step explanations for how each problem can be solved. These detailed explanations are only available to registered coaches. (pg. 56)
- **Answers to Handbook Problems:** key available to the general public. Your students can access this key, but not the full solutions to the problems. (pg. 49)
- **Problem Index + Common Core State Standards Mapping:** catalog of all handbook problems organized by topic, difficulty rating and mapping to Common Core State Standards. (pg. 53)

There are 3 types of handbook problems to prepare students for each of the rounds of the competition. You'll want to have your students practice all of these types of problems.

Warm-Ups
14 Warm-Ups in handbook
10 questions per Warm-Up
no calculators used

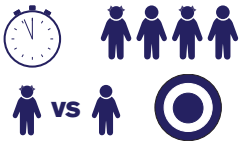
Warm-Ups prepare students particularly for the Sprint and Countdown Rounds.

Workouts
8 Workouts in handbook
10 questions per Workout
calculators used

Workouts prepare students particularly for the Target and Team Rounds.

Stretches
3 Stretches in handbook
Number of questions and use of calculators vary by Stretch

Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all 4 rounds.



IS THERE A SCHEDULE I SHOULD FOLLOW FOR THE YEAR? On average coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held before school, during lunch, after school, on weekends or at other times, coordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you're able to cover more problems and prepare your students for competitions. We've designed the *School Handbook* with this in mind. Below is a suggested schedule for the program year that mixes in Warm-Ups, Workouts and Stretches from the *School Handbook*, plus free practice competitions from last year. This schedule allows your students to tackle more difficult problems as the School and Chapter Competition approach.

Mid-August – September 2018	**October 2018**	**November 2018**	**December 2018**
Warm-Ups 1, 2 + 3 Workouts 1 + 2	Warm-Ups 4, 5 + 6 Workout 3 Measurement Stretch	Warm-Ups 7 + 8 Workouts 4 + 5 Expected Value Stretch	Warm-Ups 9, 10 + 11 Workout 6 Transformations Stretch
January 2019 Warm-Ups 12, 13 + 14 Workouts 7 + 8 *2019 MATHCOUNTS School Competition* *Select chapter competitors (optional at this time)*		**February 2019** Practice Competition: 2018 School Competition Practice Competition: 2018 Chapter Competition *Select chapter competitors (required by this time)* *2019 MATHCOUNTS Chapter Competition*	

You'll notice that in January or February you'll need to select the 1-10 student(s) who will represent your school at the Chapter Competition. This must be done before the start of your local Chapter Competition. You'll submit the names of your chapter competitors either online at **www.mathcounts.org/coaches** or directly to your local Chapter Coordinator.

It's possible you and your students will meet more frequently than once a week and need additional resources. If that happens, don't worry! You and your Mathletes can work together using the **Interactive MATHCOUNTS Platform**, powered by NextThought. This free online platform contains numerous *MATHCOUNTS School Handbooks* and past competitions, not to mention lots of features that make it easy for students to collaborate with each other and track their progress. You and your Mathletes can sign up for free at mathcounts.nextthought.com.

And remember, just because you and your students will meet once a week doesn't mean your students can only prepare for MATHCOUNTS one day per week. Many coaches assign "homework" during the week so they can keep their students engaged in problem solving outside of team practices. Here's one example of what a 2-week span of practices in the middle of the program year could look like.

Monday	Tuesday	Wednesday (Weekly Team Practice)	Thursday	Friday
-Students continue to work individually on Workout 4, due Wednesday	-Students continue to work on Workout 4 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 4 -Coach gives Warm-Up 7 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 5 as individual work, due Wednesday	-Students continue to work individually on Workout 5
-Students continue to work individually on Workout 5, due Wednesday	-Students continue to work on Workout 5 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 5 -Coach gives Warm-Up 8 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 6 as group work, due Wednesday	-Students work together on Workout 6 using online Interactive Platform

WHAT SHOULD MY TEAM PRACTICES LOOK LIKE? Obviously every school, coach and group of students is different, and after a few practices you'll likely find out what works and what doesn't for your students. Here are some suggestions from veteran coaches about what makes for a productive practice.

- Encourage discussion of the problems so that students learn from each other
- Encourage a variety of methods for solving problems
- Have students write math problems for each other to solve
- Use the **Problem of the Week** (posted online every Monday)
- Practice working in groups to develop teamwork (and to prepare for the Team Round)
- Practice oral presentations to reinforce understanding

On the following page is a sample agenda for a 1-hour practice session. There are many ways you can structure math team meetings and you will likely come up with an agenda that works better for you and your group. It also is probably a good idea to vary the structure of your meetings as the program year progresses.

MATHCOUNTS Team Practice Sample Agenda – 1 Hour

Review Problem of the Week (20 minutes)

- Have 1 student come to the board to show how s/he solved the first part of the problem.
- Discuss as a group other strategies to solve the problem (and help if student answers incorrectly).
- Have students divide into groups of 4 to discuss the solutions to the remaining parts of the problem.
- Have 2 groups share answers and explain their solutions.

Timed Practice with Warm-Up (15 minutes)

- Have students put away all calculators and have one student pass out Warm-Ups (face-down).
- Give students 12 minutes to complete as much of the Warm-Up as they can.
- After 12 minutes is up, have students hold up pencils and stop working.

Play Game to Review Warm-Up Answers (25 minutes)

- Have students divide into 5 groups (size will depend on number of students in meeting).
- Choose a group at random to start and then rotate clockwise to give each group a turn to answer a question. When it is a group's turn, ask the group one question from the Warm-Up.
- Have the group members consult their completed Warm-Ups and work with each other for a maximum of 45 seconds to choose the group's official answer.
- Award 2 points for a correct answer on questions 1-3, 3 points for questions 4-7 and 5 points for questions 8-10. The group gets 0 points if they answer incorrectly or do not answer in 45 seconds.
- Have all students check their Warm-Up answers as they play.
- Go over solutions to select Warm-Up problems that many students on the team got wrong.

OK I'M READY TO START. HOW DO I GET STUDENTS TO JOIN? Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by:
  1. posting intriguing math questions (specific to your school) and referring students to the first meeting for answers.
  2. designing a bulletin board or display case with your MATHCOUNTS poster (included in your School Competition Kit) and/or photos and awards from past years.
  3. attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
  4. adding information about the MATHCOUNTS team to your school's website.
  5. making a presentation at the first pep rally or student assembly.

Good luck in the competition! If you have any questions during the year, please contact the MATHCOUNTS national office at info@mathcounts.org.

Coach Resources: www.mathcounts.org/coaches

2018-2019 Handbook Materials

Thank you for being a coach in the MATHCOUNTS Competition Series this year!
We hope participating in the program is meaningful and enriching for you and your Mathletes.
Don't forget to log in at **www.mathcounts.org/coaches** for additional resources!

What's in This Year's Handbook

Highlighted Resources

Also access resources at www.mathcounts.org/coaches!

C Great for Coaches | M Great for Mathletes | A Advanced Level Book | Free Resource

OPLET

Online database of over 13,000 problems and over 5,000 step-by-step solutions. Create personalized quizzes, flash cards, worksheets and more!

Save $25 when you buy your subscription by Oct. 12, 2018
Renewers: use code RENEW18
First-Time Subscribers: use code NEW18

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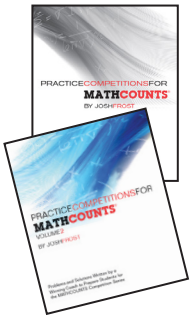
www.mathcounts.org/myoplet

Practice Competitions for MATHCOUNTS, Vol. I & II

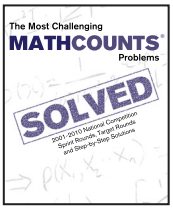


Practice books written by repeat national-level coach Josh Frost. Each volume includes 4 complete mock-competitions plus solutions.

C M

www.mathcounts.org/store

Most Challenging MATHCOUNTS Problems Solved



Advanced level practice book with 10 years of national-level Sprint Rounds, plus detailed step-by-step solutions to each problem.

C M A

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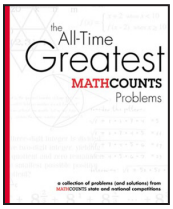
All Time Greatest MATHCOUNTS Problems



A collection of some of the most creative, interesting and challenging MATHCOUNTS competition problems.

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www.mathcounts.org/store

Interactive MATHCOUNTS Platform

Online platform of past and current handbook and competition problems. Interactive features make collaboration easy and fun!

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mathcounts.nextthought.com

MATHCOUNTS Trainer App

Train your Mathletes with this fun app, featuring real-time leaderboards and lots of past MATHCOUNTS problems.

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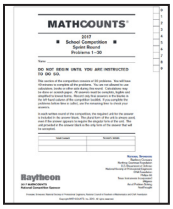
aops.com/mathcounts_trainer or download at the App Store

Past Competitions



Last year's School, Chapter and State competitions are free online! Other years' competitions can be purchased.

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www.mathcounts.org/pastcompetitions
www.mathcounts.org/store

Problem of the Week

A new, multi-step problem every week! Each problem focuses on a particular set of math skills and coincides with a timely event, holiday or season. Get the problem at the beginning of the week and the step-by-step solution the following week.

C M

www.mathcounts.org/potw

MATHCOUNTS Minis

A fun monthly video series featuring Richard Rusczyk from Art of Problem Solving. Each video looks at a particular math skill and walks through how to solve different MATHCOUNTS problems using creative problem-solving strategies.

C M

www.mathcounts.org/minis

Critical 2018-2019 Dates

2018

Aug. 15 – Dec. 14

Submit your school's registration to participate in the Competition Series and receive this year's School Competition Kit, which includes a hard copy of the *2018-2019 MATHCOUNTS School Handbook*. Kits are shipped on an ongoing basis between mid-August and December 31.

The fastest way to register is online at **www.mathcounts.org/compreg**. You also can download the MATHCOUNTS Competition Series Registration form and mail or email it with payment to:

MATHCOUNTS Foundation – Competition Series Registrations
1420 King Street, Alexandria, VA 22314
Email: reg@mathcounts.org

To add students to your school's registration, log in at **www.mathcounts.org/coaches** to access the Dashboard. **Questions?** Email the MATHCOUNTS national office at info@mathcounts.org.

Nov. 2

The 2019 School Competition will be available online. All registered coaches can log in at www.mathcounts.org/coaches to download the competition.

Nov. 2 (postmark)

Deadline to register for the Competition Series at reduced registration rates ($30 per student, $300 for full registration of 10 students). After November 2, registration rates will be $35 per student, $350 for full registration.

Dec. 14 (postmark)

Competition Series Registration Deadline
In some circumstances, late registrations might be accepted at the discretion of MATHCOUNTS and the local coordinator. *Late fees will apply. Register on-time to ensure your students' participation.*

2019

Early Jan.

If you have not been contacted with details about your upcoming competition, call your local or state coordinator. Coordinator contact information is available at www.mathcounts.org/findmycoordinator.

Late Jan.

If you have not received your School Competition Kit, contact the MATHCOUNTS national office at info@mathcounts.org.

Feb. 1-28 — **Chapter Competitions**

March 1-31 — **State Competitions**

May 12-13 — **2019 Raytheon MATHCOUNTS National Competition in Orlando, FL**

Measurement Stretch

1. _____ units

Merri places weights of 6 units and 28 units on the right side of a balance and weights of 3 units and 19 units on the left side. If she adds an object to the left side that makes the balance level, how many units does the object weigh?

2. _____ tacks

The weight of a small clip is $\frac{2}{3}$ the weight of a large clip. If 2 tacks weigh the same amount as a large clip, how many tacks weigh the same amount as 12 small clips?

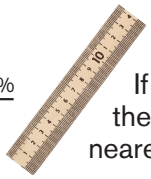
3. _____ Dems

On the planet Klem, 1 Bem plus 7 Dems equals 4 Pems, and 2 Bems plus 1 Dem equals 1 Pem. How many Dems equal 7 Bems?

4. _____ meters

If a race car is traveling at 99 mi/h, how many meters does it travel in a second, given that 0.305 meter = 1 foot? Express your answer as a decimal to the nearest tenth.

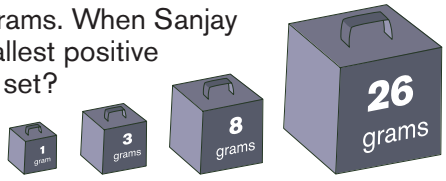
5. _____ %



If the results when reading a measuring stick can be off by at most 1 cm, what is the maximum percent error when 24 cm is measured? Express your answer to the nearest tenth.

6. _____ grams

Vijay gives Sanjay a set of four weights of 1, 3, 8 and 26 grams. When Sanjay places weights on either side of a balance, what is the smallest positive integer number of grams that he **cannot** measure with this set?



7. _____ gallons

If Clem has 2 cups, 7 pints, 8 quarts and 11 half-gallons of lemonade, how many total gallons of lemonade does she have? Express your answer as a mixed number.

8. _____ Klegs

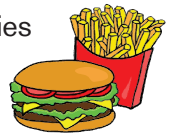
If 2 Blams equal 15 Droms and 5 Droms equal 28 Klegs, how many Klegs are in a Blam?

9. _____

What is the ratio of 1 ounce to 1000 grams, given that 1 pound equals 454 grams? Express your answer as a decimal to the nearest thousandth.

10. _____ times

If one order of fries and five burgers cost twice as much as three orders of fries and two burgers, how many times as much does a burger cost compared to one order of fries?



Expected Value Stretch

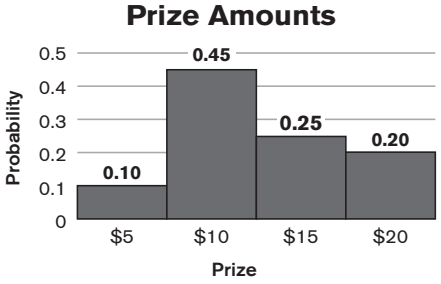
If the outcomes of random variable X have values x_1, x_2, x_3, ..., x_n and the probabilities of these outcomes occurring are p_1, p_2, p_3, ..., p_n, respectively, then the **expected value** of the outcome is the sum of the products of the probability of each outcome and the value of that outcome.

$$E(X) = p_1 x_1 + p_2 x_2 + p_3 x_3 + \cdots + p_n x_n$$

11. _____ An unfair six-sided die with faces labeled 1, 2, 3, 5, 8 and 13 is rolled. The table lists the probability of the die landing with each number showing on the top face. The expected value of the roll is the sum of the products of each face value and its corresponding probability of being rolled. What is the expected value when the die is rolled? Express your answer as a mixed number.

Top Face Value	Probability
1	$\frac{1}{3}$
2	$\frac{1}{15}$
3	$\frac{1}{6}$
5	$\frac{1}{5}$
8	$\frac{2}{15}$
13	$\frac{1}{10}$

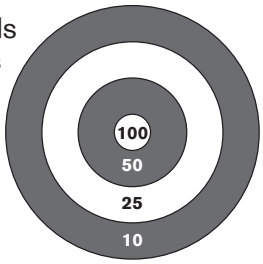
12. $_____ Terry plays a game with prizes of 5, 10, 15 and 20 dollars. The graph shows each possible prize amount and its corresponding probability. The expected value of her prize is the sum of the products of each prize and the probability of winning that prize. What is the expected value of Terry's prize?



13. _____ A fair 10-sided die with one face labeled 1, two faces labeled 2, three faces labeled 3 and four faces labeled 4 is rolled. What is the expected value when this die is rolled?

14. _____ cm^2 Ana has a bowl containing two square tiles, one with side length 2 cm and the other with side length 3 cm. She randomly chooses a tile from the bowl. The expected value of the area of the chosen tile is the sum of the products of each tile's area and its corresponding probability of being chosen. If the probability of choosing a particular tile is proportional to its area, what is the expected value of the area of the tile Ana chooses? Express your answer as a common fraction.

15. _____ points For the dartboard shown, the number of points scored when a dart lands in each region is indicated. The innermost circle of the board has radius 1 inch, and each subsequent circle has a radius 2 inches greater than the previous circle. Kane throws a dart that lands randomly somewhere on the board. What is the expected value of the number of points he scores? Express your answer as a decimal to the nearest tenth.



16. _____ Gwen randomly draws a card from a deck of 40 cards numbered 1 through 40. What is the expected value of the number on the card she draws? Express your answer as a decimal to the nearest tenth.

17. _____ faces Luke paints each face of a 5 × 5 × 5 cube red. He then cuts the cube into 125 unit cubes and randomly chooses a single unit cube. What is the expected value of the number of painted faces on this unit cube? Express your answer as a decimal to the nearest tenth.

A property of E is that it is a linear function of the random variable. So, for random variables X and Y, the expected value of the sum of random variables equals the sum of their expected values.

$$E(X + Y) = E(X) + E(Y)$$

18. _____ points In each round of a particular game, Dinara can win at most one point. If she has a 70% chance of winning a point in each round, what is the expected value of Dinara's total score after three rounds? Express your answer as a decimal to the nearest tenth.

19. _____ Jo and her four friends each secretly pick a random integer from −5 to 5, inclusive. What is the expected value of the sum of the five chosen numbers?

20. _____ jelly beans Allen randomly distributes 1000 jelly beans into 10 jars lined up in a row from left to right. What is the expected value of the number of jelly beans in the leftmost jar?

Transformations Stretch

21. _____ units A point P(−3, 2) is translated right 4 units to its image P′. The point P′ is then translated up 3 units to its image P″. What is the distance from P to P″?

22. _____ units A segment has endpoints A(0, 0) and B(−3, 4). Point C is the image of point B translated down 4 units and left 3 units. What is the perimeter of $\triangle ABC$?

23. (,) A point Q(−3, 4) is reflected across the x-axis, and then the image Q′ is reflected across the line $x = 2$. What are the coordinates of the image Q″? Express your answer as an ordered pair.

24. _____ A point S(1, 6) is reflected across the line $x - 2y = -6$. What is the sum of the coordinates of the image S′?

25. (,) What are the coordinates of the image of point D(−5, −3) when it is rotated 90 degrees clockwise about the origin? Express your answer as an ordered pair.

26. (,) What are the coordinates of the image of the point E(3, −1) when it is rotated 90 degrees counterclockwise about the point F(5, 4)? Express your answer as an ordered pair.

27. _____ A segment with endpoints G(−2, 3) and H(4, 7) is dilated by a scale factor of $\frac{2}{3}$ with center of dilation (0, 0). What is the sum of all the coordinates of G′ and H′?

28. _____ Point J(4, 8) is dilated by a scale factor of $\frac{3}{2}$ with center of dilation K(2, 2). What is the product of the coordinates of J′?

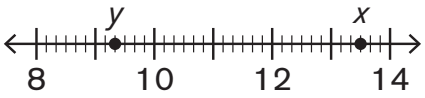
29. _____ units^2 A point L(−2, 4) is rotated 90 degrees clockwise about the point M(3, 2). Point N is the image of L′ dilated by a scale factor of $\frac{3}{2}$ with center of dilation M. What is the area of $\triangle LMN$? Express your answer as a common fraction.

30. _____ units A point R(−5, 3) is reflected across the line $y = x - 2$, and then the image R′ is rotated 90 degrees clockwise about the origin. What is the distance from R to R″? Express your answer in simplest radical form.

Warm-Up 1

31. _____ combinations Bob has 40 cents in his pocket. If Bob has no pennies, how many different combinations of quarters, dimes and/or nickels could he have?

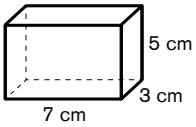
32. _____ On the number line shown, what is the value of $x - y$? Express your answer as a mixed number.



33. _____  Ted flips a coin that is equally likely to land heads up or tails up. Ted flips the coin 10 times, and each time it lands heads up. What is the probability that the next flip will also land heads up? Express your answer as a common fraction.

34. _____ What is the value of $9 + 5 \times 3 - 8 \div 2$?

35. _____ If two more than three times x is equal to five less than ten times x, what is the value of x?

36. _____ cm^3 

What is the volume of a rectangular prism of height 5 cm, width 7 cm and depth 3 cm?

37. _____ What is the average of the prime numbers between 20 and 30?

38. _____ lines How many lines of symmetry does an isosceles right triangle have?

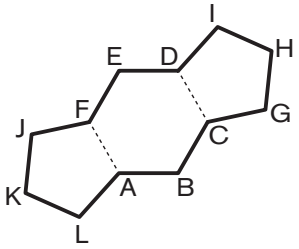
39. _____ What is the quotient when 1,000,000,000 is divided by $2^8 \times 5^7$?

40. _____ people Of 1000 people surveyed, one-third of the 630 people who reported owning a cat also own a dog. If each person surveyed owns a cat, a dog or both, how many own a dog?

Warm-Up 2

41. _____ If the value of x is 10, what is the value of $3x + 4x + 5x$?

42. _____ feet In the figure, two regular pentagons have been attached to a regular hexagon to create a 12-gon. If each regular polygon has side length 3 feet, what is the perimeter of the resulting 12-gon?



43. _____ ft/s A speed of 60 miles per hour is equal to 88 feet per second. If the speed limit in a school zone is 15 miles per hour, what is the speed limit in this zone in feet per second?

44. _____ divisors How many divisors of 64 are perfect squares?

45. _____ If $ab = 2c$, $bd = c$, $b \neq 0$ and $d = 16$, what is the value of a?

46. _____ square blocks

All of the streets in Tom's city are on a regular rectangular grid and run east-west or north-south. Tom starts at the intersection of Poyntz Avenue and Eleventh Street and walks 2 blocks east, then 3 blocks north, then 4 blocks east, then 5 blocks north. Tom then returns to his starting location by walking 6 blocks west, then 8 blocks south. What is the area, in square blocks, enclosed by Tom's path?

47. _____ What is the value of $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}-\frac{1}{5}}$? Express your answer as a common fraction.

48. _____ What is the arithmetic mean of $\frac{2}{3}$, $\frac{7}{9}$, $\frac{1}{4}$ and $\frac{5}{16}$? Express your answer as a common fraction.

49. _____ If a and b are real numbers such that $a + b = a - b$ and $a \neq b$, what is the value of $\dfrac{a^2b + a + b - ab^2}{a - b}$?

50. _____ Given that 75% of a certain number is 88, what is $\frac{3}{8}$ of the number?

Warm-Up 3

51. _____ jelly beans All the jelly beans in Sammi's cup are red, yellow or orange. Five are red and seven are orange. If two-thirds are yellow, how many jelly beans are there in Sammi's cup?

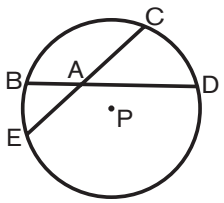
52. _____ What is the value of the sum 0.49 + 0.53 + 0.55 + 0.47 + 0.48? Express your answer as a decimal to the nearest hundredth.

53. _____ outfits If an outfit consists of jeans, a shirt, a sweater and a scarf, how many different outfits can Allie make from one pair of jeans, four shirts, three sweaters and two scarves?

54. _____ minutes Destiny can run one-eighth of a mile in three-quarters of a minute, and she can walk one-quarter of a mile in 4 minutes. How many minutes total will it take for Destiny to run one mile and then walk one mile?

55. _____ What is the value of $27^2 - 23^2$?

56. _____ units In circle P, two chords intersect at A, as shown, with AB = 3 units, AC = 8 units and AD = 6 units. What is the value of AE? Express your answer as a mixed number.



57. _____ days Six people are invited to attend a five-day conference scheduled for Monday through Friday. Grace will attend only on Monday and Thursday. Becca will attend every day except Monday. Carmen will attend every day except Thursday. Davis will attend only on Thursday. Ernie will only attend either Thursday or Friday. Frank will attend every day except Monday and Wednesday. On how many days during the conference will at least three of these six people be in attendance?

58. _____ miles  The bus from Kevin's home to the middle school travels 8 miles west, then turns and travels 8 miles north, then turns and travels 7 miles west to arrive at school. If the bus were able to travel directly from Kevin's house to the middle school along a straight path, how much shorter would the trip be?

59. _____ When twenty-one-and-a-half trillion is written in scientific notation, what is the exponent needed on the base of 10?

60. _____ diagonals If a polygon has 17 sides, how many diagonals does it have?

Warm-Up 4

61. _____ degrees What is the degree measure of the complement to an angle that is a supplement to an angle of measure 163 degrees?

62. _____ If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, what is the value of x when $\begin{vmatrix} x & 4 \\ 3 & 10 \end{vmatrix} = 38$?

63. _____ : a.m. Every morning, the Sharetrain arrives in Mountain View at 9:19 a.m. It takes Miranda between 17 and 21 minutes to walk to the train station from home. If she wants to guarantee that she will arrive at the station with at least 5 minutes to spare, what is the latest time she can leave home?

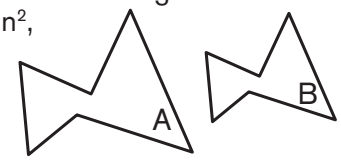
64. _____ Ryan picks two different numbers from the set {2, 3, 5, 7} and multiplies them. What is the absolute difference between the greatest and the least products that Ryan can get?

65. $ _____ Zu's zoo offers a promotional deal: get a free $3 cotton candy and a free $2 soda with the purchase of five $12 admission tickets. A group of 20 students will each purchase an admission ticket. If half of them want cotton candy and one-fourth of them want soda, how many dollars would they save by using the promotional deal?

66. _____ ounces Preston has four potatoes, each of which weighs a whole number of ounces. The median weight of his potatoes is 11 ounces, and the mean weight of his potatoes is 12 ounces. What is the greatest possible difference between the weight of the heaviest and lightest of his potatoes?

67. _____ mm Photographers often rely on a rule to choose a shutter speed in seconds that is the reciprocal of the effective focal length of the lens in millimeters. If Jackie is shooting at a focal length of 80 mm, Clarise is shooting at 200 mm, Sage is shooting at 400 mm, and they all apply this reciprocal rule, what focal length corresponds to the sum of their shutter speeds?

68. _____ in^2 Hexagons A and B are geometrically similar. The shortest sides of the two hexagons are 4 inches and 3 inches, respectively. If the area of hexagon A is 48 in^2, what is the area of hexagon B?



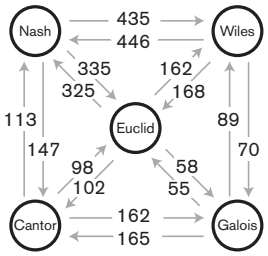
69. _____ What is the value of $1 \times 12 + 2 \times 11 + 3 \times 10 + 4 \times 9 + 5 \times 8 + 6 \times 7 + 7 \times 6 + 8 \times 5 + 9 \times 4 + 10 \times 3 + 11 \times 2 + 12 \times 1$?

70. _____ degrees What is the measure of an interior angle of a regular polygon with 90 sides?

Warm-Up 5

71. $\$$_____ For each lawn Kayla mows, she charges a flat fee of \$15, plus an amount proportional to the area of the lawn. If Kayla charges the owner of a 20-foot by 40-foot lawn \$24, how much would she charge the owner of a 60-foot by 80-foot lawn?

72. $\$$_____ Alicia needs to make her way from Nash to Wiles. Available flights and their respective costs, in dollars, are indicated in the figure shown. If Alicia can take any sequence of flights, what is the total cost of her least expensive route from Nash to Wiles?



73. _____ Ancient Egyptians used the symbols shown in Figure 1 to express certain quantities by placing the symbols in any order. What quantity is represented by the symbols in Figure 2?

Staff	Heel	Rope	Lotus	Finger
1	10	100	1000	10,000

Figure 1

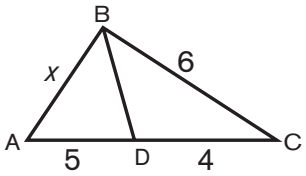
Figure 2

74. _____ Yixin thinks of a positive number that is a perfect cube, and Aly thinks of a number that is a perfect square. If the sum of their numbers is 31, what is the product of their numbers?

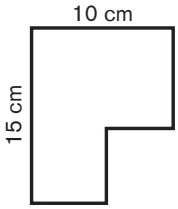
75. _____ integers Jalacia erases some integers from the following list: 5, 9, 2, 3, 7, 8, 6, 5, 6, 4, 1, 7, 9, 8. When she is done, the range of the remaining list is 2. What is the least possible number of integers that Jalacia could have erased?

76. $\$$_____ Esther purchased two rock tumblers and one spy pen for \$74. Eli purchased two puzzles and a spy pen for \$50. Sabine purchased a rock tumbler and two puzzles for \$57. Based on this, how much does one puzzle cost?

77. _____ cm In triangle ABC, shown here, segment BD bisects angle ABC. If CD = 4 cm, AD = 5 cm and BC = 6 cm, what is the value of AB? Express your answer as a common fraction.



78. _____ cm



In the figure shown, two sides have lengths 10 cm and 15 cm. If all line segments intersect at right angles, what is the perimeter of the figure?

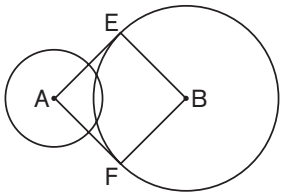
79. _____ If $a = 2 + \frac{3}{4+\frac{5}{6}}$ and $b = 6 + \frac{5}{4+\frac{3}{2}}$, what is the value of $\frac{a}{b}$? Express your answer as a common fraction.

80. _____ sides If the difference between the measures of an interior angle and an exterior angle of a regular polygon is 100 degrees, how many sides does the polygon have?

Warm-Up 6

81. _____ inches When Jim was 10 years old, he was 4 feet 8 inches tall. When he was 20 years old, Jim was 6 feet 2 inches tall. By how many inches did Jim's height increase in those 10 years?

82. _____ Circles A and B intersect, and points E and F are on circle B as shown. If quadrilateral AEBF is a square, what is the probability that a randomly chosen line through point A intersects both circles? Express your answer as a common fraction.



83. _____  Lauren randomly picks two cards, without replacement, from a group of six cards numbered 2, 3, 5, 6, 7 and 10. What is the probability that the product of the two numbers on the selected cards is a multiple of 10? Express your answer as a common fraction.

84. _____ Alana draws four squares, five pentagons and six octagons. Marie draws n hexagons and notices that the combined number of diagonals in her hexagons is equal to the combined number of diagonals in Alana's polygons. What is the value of n?

85. _____ Consider the set of all four-digit positive integers less than 2000 whose digits have a sum of 24. What is the median of this set of integers?

86. _____ If $x^2 - y^2 = 7$, $x = \frac{12}{y}$ and $y < 0$, what is the value of $x^4 + y^4$?

87. $ _____ At the county fair, two hot dogs and an ice cream cone cost $2.50. Two slices of pizza and an ice cream cone cost $3.50. What is the total cost of one hot dog, one slice of pizza and one ice cream cone at the county fair?

88. _____ When the positive integer divisors of 385 are arranged from least to greatest, what is the sum of the 4th, 5th and 6th divisors?

89. _____ minutes Gavin goes for a run at a constant pace of 9 minutes per mile. Ten minutes later, Lars goes for a run, along the same route, at a constant pace of 7 minutes per mile. How many minutes does it take for Lars to reach Gavin?

90. _____ degrees Lines are drawn from the center of a 72-gon to two of its vertices. The smaller central angle formed by these lines includes seven sides of the polygon. What is the degree measure of this central angle?

Warm-Up 7

91. _____ problems Two days ago, Neil was assigned a set of problems to solve. Today he solved the final three. Yesterday he solved half of those then remaining plus half a problem. The first day he solved half of those assigned plus half a problem. How many problems were in the assigned set?

92. _____ scores A deck of 14 cards numbered 1 through 14 is dealt to Ken and Gunther so that each gets 7 cards. Each player's score is the sum of his card values, and the player with the lower score wins. How many different winning scores are possible?

93. _____ What is the units digit of the sum $1! + 2! + 3! + \cdots + n!$ when $n = 2019$?

94. _____ inches

The sum of the heights of Alex and his two brothers, Evan and Joel, is 221 inches. Alex is 8 inches taller than Evan and 5 inches taller than Joel. What is Alex's height, in inches?

95. _____ If all test scores are integers from 0 to 100, inclusive, what is the least possible median of five test scores that add to 204?

96. _____ If $f(x) = \dfrac{4-x^2}{x+2}$ and $g(x) = x - 2$, what is the value of $\dfrac{f(x)}{g(x)}$, for $x \neq \pm 2$?

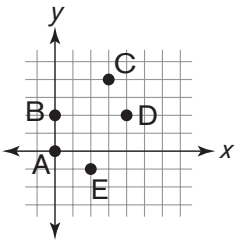
97. _____ jumps

Freddie Frog traveled 29 yards in 261 jumps, at a constant distance per jump. After hurting a toe, her distance per jump decreased by 1 inch. How many jumps more than 261 will it take for her to travel another 29 yards but with a hurt toe?

98. _____ On the first day of class, Smiley's teacher gave her a piece of paper containing Smiley's seven-digit locker combination. Smiley noticed that the seven-digit number was a multiple of 11. When she went to her locker, she saw that one digit on the paper had been completely smudged, as shown. Smiley had forgotten the smudged digit. What was Smiley's seven-digit locker combination?

99. _____ ways How many ways are there to put twelve identical donuts into three differently colored boxes if there must be at least two donuts in each box?

100. _____ units^2 On a coordinate grid, Micah constructs a pentagon with vertices A(0, 0), B(0, 2), C(3, 4), D(4, 2) and E(2, −1). What is the area of pentagon ABCDE?



Warm-Up 8

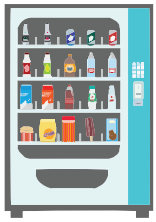
101. _____ kg Don's signature coffee blend is 60% dark roast and 40% light roast. He has 10 kg of blend A, which is 80% dark roast and 20% light roast, and 10 kg of blend B, which is 20% dark roast and 80% light roast. How many kilograms of blend A will Don need to use to make 10 kg of his signature blend? Express your answer as a mixed number.

102. _____ minutes Coordinated Universal Time (UTC) is 5 hours later than Eastern Standard Time (EST). If Nico and Jon start playing a tennis match in London at 8:00 a.m. UTC and finish at 12:15 p.m. EST, for how many minutes did they play?

103. _____ times While working on a history project, Erika writes down a list of years starting with 1809 and ending with 2019. How many times did Erika write the digit 1?

104. _____ What is the probability that a sequence of five flips of a fair coin will not land heads up twice in a row? Express your answer as a common fraction.

105. _____ games Liz, Eva and Ace have played trivia as a team 33 times and won 24 times. What is the minimum number of games they must win to have an overall winning percentage of 80%?

106. _____ ways 

A certain vending machine accepts nickels, dimes, quarters and dollar bills, and it provides change using nickels, dimes and quarters. If Sarah selects a healthful snack priced at $1.30 after inserting $2.00, in how many ways can the vending machine provide Sarah's change?

107. _____ What is the geometric mean of the median and mode of the set {23, 25, 3, 25, 20, 22, 21, 2, 1, 14, 12}? Express your answer in simplest radical form.

108. _____ What is the mean of all three-digit positive integers whose digits are in the set {2, 0, 1, 9}?

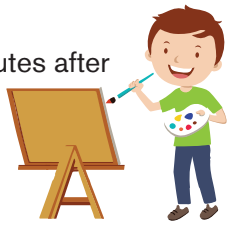
109. _____ If we define the binary operation ⌘ as $a \mathbin{⌘} b = ab + b$ for all numbers a and b, what positive value of x satisfies the equation $x \mathbin{⌘} (4 \mathbin{⌘} x) = 550$?

110. _____ degrees A circle with center P is inscribed in isosceles triangle ABC with apex angle A measuring 34 degrees. What is the degree measure of angle APC? Express your answer as a decimal to the nearest tenth.

Warm-Up 9

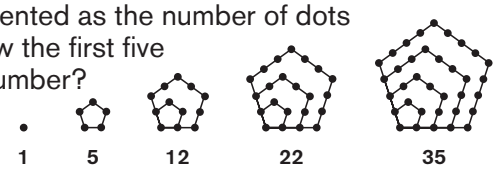
111. _____ points Thomas is a basketball player who is x inches tall. One season he averaged y points per game. The sum of x and y is 98, and x is 11 more than twice y. Based on these statistics, how many points did Thomas score in the 60 games he played that season?

112. _____ : _____ p.m. Pierre needs 39 minutes to paint a painting. The painting will be dry 55 minutes after Pierre finishes. If Pierre starts painting at 1:00 p.m., what is the earliest time when he can have three finished, dry paintings?



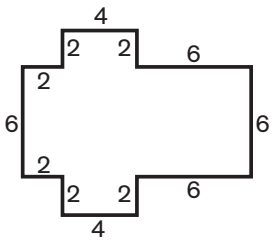
113. _____ pounds Blackbeard and Redbeard take a total of 60 pounds of gold from Treasure Cove. Because Blackbeard outranks Redbeard, Blackbeard gets $\frac{5}{8}$ of the gold and Redbeard gets the remaining $\frac{3}{8}$. How many more pounds of gold does Blackbeard get than Redbeard?

114. _____ A pentagonal number is a number that can be represented as the number of dots on a regular pentagon as in these figures, which show the first five pentagonal numbers. What is the 20th pentagonal number?



115. _____ What is the arithmetic mean of all 3-digit numbers whose digits are distinct and nonzero?

116. _____



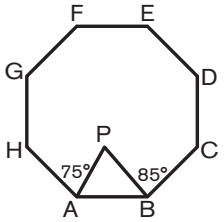
The figure shown can be folded into a rectangular prism. What is the absolute difference between the numerical values of the surface area and volume of the prism?

117. _____ If the sum of three numbers is equal to their product, and two of the numbers are $\frac{4}{3}$ and 1, what is the third number?

118. _____ Alvin plays mini-golf with four friends. The median and unique mode of all their scores are both 10, and the range of their scores is 16. If the mean of all their scores is 15, what is the sum of all possible values of the lowest score?

119. _____ Cayley has a fair six-sided die whose faces are numbered −2, −1, 0, 0, 1 and 2. She rolls the die three times. What is the probability that the sum of the three numbers she rolls is 0? Express your answer as a common fraction.

120. _____ degrees Triangle ABP is drawn inside a regular octagon as shown. What is the degree measure of acute angle P if $m\angle HAP = 75$ degrees and $m\angle CBP = 85$ degrees?



Warm-Up 10

121. _____ If $\bigcirc(x) = 2x + 1$ and $\triangle(x) = 2x - 1$, what is the value of $\bigcirc(\bigcirc(\pi)) - \triangle(\triangle(\pi))$?

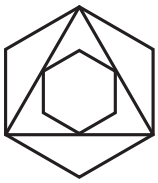
122. _____ What is the sum of the two values of x for which $|x - 3| + |x - 7| = 6$?

123. _____ units^2 What is the area of triangle ABC, with vertices A(6, 8), B(9, 2) and C(17, 6)?

124. _____ students A local college has 985 students, of whom 460 play a varsity sport and 571 belong to a club. What is the absolute difference between the least and greatest possible numbers of students who play a varsity sport AND belong to a club?

125. _____ in^2 What is the smallest possible area of an isosceles triangle with side lengths 5 inches and 6 inches? Express your answer as a decimal to the nearest tenth.

126. _____ An equilateral triangle is inscribed in a regular hexagon, and a smaller regular hexagon is inscribed inside the triangle so that three of its vertices are each the midpoint of a side of the triangle, as shown. What is the ratio of the area of the smaller hexagon to that of the larger hexagon? Express your answer as a common fraction.

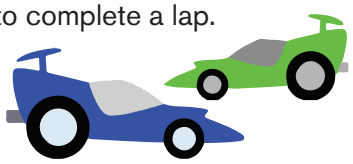


127. _____ divisors How many positive integer divisors of 23,328 are perfect cubes?

128. _____ points Saila took nine exams, each scored out of 100 points. She received a passing score of 60 or above on each of them. If the mean of her nine scores was 80 points, what is the greatest possible value of the median of her nine scores?

129. _____ paths If A(0, 0, 0) and B(2, 2, 2) are points in coordinate space, how many paths are there from A to B that move from one lattice point to another in the positive x-, y- or z-direction?

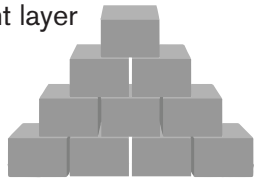
130. _____ seconds Two race cars travel in opposite directions in separate lanes of a circular track. Car F takes 120 seconds to complete a lap. Car S takes 240 seconds to complete a lap. Once the two cars pass each other, how long does it take for them to pass each other again?



Warm-Up 11

131. _____ What is the greatest common divisor of 2563 and 4147?

132. widgets Hongyi is making a triangular display of widgets. The bottom layer will contain n widgets. The layer immediately above it will contain $n - 1$ widgets. Each subsequent layer will contain one fewer widget than the previous layer, and the top layer will contain one widget. If widgets are packed 25 per case, and Hongyi must use all of the widgets in every case he opens, what is the smallest positive number of widgets Hongyi can use to build the display?

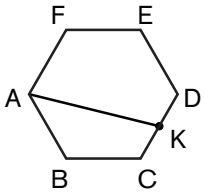


133. degrees Regular pentagon ABCDE has diagonal BE. If P is the midpoint of side ED, what is the degree measure of angle EBP?

134. prime dates A *prime date* is a date for which the number of the month and the number of the day are both prime numbers. How many prime dates are there in the month of March?

135. miles Bennie and Flossie are traveling on a straight road, going in the same direction and starting at the same place. Bennie is traveling at an average speed of 62 mi/h, and Flossie is traveling at an average speed of 75 mi/h. After 2 hours, Flossie stops for a 20-minute break before continuing. After a total of 5 hours, what is the distance between them?

136. cm



Regular hexagon ABCDEF has side length 4 cm. Point K is the midpoint of side CD. What is the length of AK? Express your answer in simplest radical form.

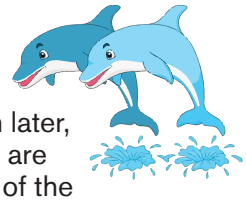
137. crackers At every lunch, three friends share their cheesy crackers with each other but agree not to eat them until they all have the same number. At the first lunch, Axel gives Ben and Chloe as many crackers as they each already have. At the second lunch, Ben gives Axel and Chloe as many crackers as they each already have. At the third lunch, Chloe gives Axel and Ben as many crackers as they each already have, after which they each have 8 crackers, so they eat them. How many crackers did Axel start with?

138. sequences

Paul and his friends play a version of tag football in which a team earns 5 points for every touchdown and 3 points for every field goal. If Paul's team has a total of 25 points, how many different sequences of field goals and touchdowns could the team have scored?

139. _____ Five numbers form an arithmetic sequence with a mean of 18. If the mean of the squares of the five numbers is 374, what is the greatest of the five original numbers?

140. dolphins An oceanographer tags every dolphin that she sees over a one-week period. At the end of the week, she has tagged 1000 dolphins. One month later, when she returns and examines 2000 dolphins, she finds that 400 of them are ones that she tagged. Assuming that each time, she saw a random subset of the local dolphin population, what is the expected number of dolphins in the local population?



Warm-Up 12

141. _____ cubes Exactly 512 small cubes perfectly fill a lidless cubical box. All of the small cubes are removed except those touching the bottom of the box and those touching the sides of the box. How many of the small cubes were removed?

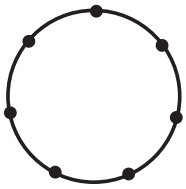
142. _____ To earn Gold Status on a social media site, Max needs 800 points. He earns a point for every 3 posts he reads, and he gets 15 points for each post he writes. Max can write at most 3 posts a day. He can read up to 120 posts on a weekday and 225 posts on a weekend day. If he starts with 0 points on a Monday morning, what day of the week is the day on which he can first earn Gold Status?

143. _____ If $\frac{\frac{a}{4}}{3} = \frac{4}{\frac{a}{3}}$, what is the value of a^2?

144. _____ cm Segment AC is a diameter of circle P, which has radius 13 cm. If B is a point on circle P such that AB = 10 cm, what is the length of segment BC?

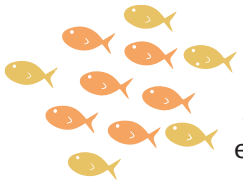
145. _____ integers How many positive integers are each a divisor of both 630 and 360 but not a divisor of 60?

146. _____ pentagons Seven points are equally spaced around a unit circle. How many non-congruent convex pentagons can be drawn by using a subset of the points as vertices of the pentagons?



147. _____ If the value of a is 25 and $a - b = \frac{a}{b} - 1$, what is the sum of all possible values of b?

148. _____ If square WXYZ has vertices W(2, 1), X(a, b), Y(4, 7) and Z(c, d), what is the value of $|ab - cd|$?

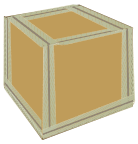
149. _____  The ten guppies in Vida's tank are 5, 5, 8, 8, 8, 9, 10, 11, 11 and 33 weeks old. A guppy of age n weeks is added to her tank. The sum of the mean, median and unique mode of the eleven guppies' ages equals the range. What is the sum of all possible integer values of n?

150. _____ cm If a rhombus with area 26 cm^2 has one diagonal of length 4 cm, what is the length of its other diagonal?

Warm-Up 13

151. _____ What is the coefficient of abc when the product $(a + 2b)(b + 2c)(c + 2a)$ is expanded and like terms are combined?

152. _____ candies Jenny has a bag that contains equal numbers of red, green, yellow, orange and purple candies. She likes only the red candies. If she randomly selects a candy from her bag and discards any non-red candy she selects, then she is guaranteed to select a red candy within a minimum of 45 random selections. What is the total number of candies in her bag?

153. _____ feet  The dimensions of a packing box are 15 inches by 13 inches by 10 inches. Every edge of the box is to be secured with packing tape. If the length of tape along each edge is equal to the edge length, how many feet of tape are needed? Express your answer as a mixed number.

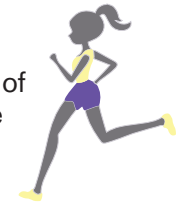
154. _____ units What is the length of the hypotenuse of a right triangle with legs of length $12\sqrt{3}$ units and $23\sqrt{3}$ units? Express your answer in simplest radical form.

155. _____ people It takes a crew of eight people three days to paint an aircraft. At this rate, how many people are needed to paint five aircraft in six days? 

156. _____ In a particular sequence, the ratio between consecutive terms remains constant. If the 2nd term is 2, and the 5th term is 5, what is the value of the 11th term of the sequence? Express your answer as a common fraction.

157. _____   Ilana rolls a fair 8-sided die with faces labeled 1 through 8, and Yolanda rolls a fair 12-sided die with faces labeled 1 through 12. What is the probability that Ilana rolls a smaller number than Yolanda? Express your answer as a common fraction.

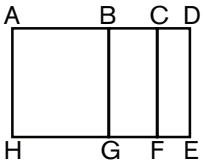
158. _____ units Rectangle PQRS is drawn on a coordinate plane. The y-coordinate of Q is 24 units greater than the y-coordinate of S, the x-coordinate of R is 25 units greater than the x-coordinate of P, and the y-coordinate of P is equal to the y-coordinate of R. What is the perimeter of rectangle PQRS?

159. _____ miles Hannah ran five races, each of which was a different positive integer number of miles. The mean of her distances was 4 miles. What is the maximum possible distance of her longest race? 

160. _____ If the coefficients c and d of $x^3 - 19x^2 + cx + d = 0$ are chosen so that the roots of the equation are positive integers that form a geometric sequence, what is the value of $c + d$?

Warm-Up 14

161. _____ units Rectangles ABGH, BCFG and CDEF are adjacent, as shown, with AH = n, AB = $n + 3$, BC = $n - 3$ and CD = $n - 6$. If rectangle BDEG has area 110 units^2, what is the perimeter of rectangle ACFH?



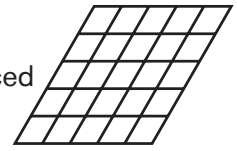
162. _____ units The larger of two right triangles has area numerically equal to 3 times the sum of its integer leg lengths, a and b. The other right triangle has area numerically equal to 2 times the sum of its integer leg lengths, c and d. If the triangles have areas in the ratio 3:2, what is the value of $a + b + c + d$?

163. _____ The vertices of a cube are labeled with the integers 1 through 8, in some order, with each integer used exactly once. Each edge of the cube is labeled with the product of the integers at its two endpoints. Each face of the cube is labeled with the sum of the labels of its four edges. What is the greatest possible absolute difference between the labels of two adjacent faces of the cube?

164. _____ Burt has a set of tiles spelling BANANAS. Ernie randomly chooses a set of five tiles from Burt's set. What is the probability that Ernie can create a palindrome using all five tiles? Express your answer as a common fraction.

165. _____ Screen sizes of rectangular television sets are measured on the diagonal. If Joe owns a television with a 62-inch screen, and Daniel owns a geometrically similar television with a 31-inch screen, what is the ratio of the area of Daniel's screen to the area of Joe's screen? Express your answer as a common fraction.

166. _____ rhombi How many rhombi of any size are in this figure, composed of equally spaced horizontal lines and parallel diagonal lines that are also equally spaced?



167. _____ What is the 15th digit to the right of the decimal point in the decimal expansion of $\frac{3}{7}$?

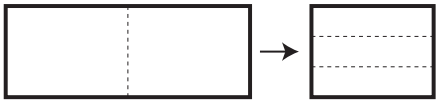
168. _____ If 6, a, b, c, d, e, 34 is a list of seven different positive integers written in increasing numerical order such that c is both the mean and the median of the list, what is the least possible value for c?

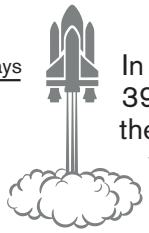
169. _____ Two real numbers from 0 to 10, inclusive, are to be chosen at random. What is the probability that the absolute difference between the two numbers will be less than 3? Express your answer as a common fraction.

170. _____ permutations How many permutations of the digits 1, 2, 3 and 4 can be arranged in a line, so that no adjacent digits differ by more than two?

Workout 1

171. _____ cm A rectangular sheet of paper is cut in half perpendicular to the longer side. One half is discarded, and the other half is cut into thirds as shown. Two of the thirds are discarded, and the remaining third is cut into fourths by vertical lines. Three of the fourths are discarded, and the remaining fourth is cut into fifths by horizontal lines. Four of the fifths are discarded, and the remaining fifth is cut into sixths by vertical lines. Five of the sixths are discarded, leaving a square piece with side length 2 cm. What is the perimeter of the original sheet of paper?



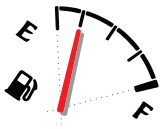
172. _____ days  In 1969, the Apollo 10 mission set the record for the fastest crewed space travel, at 39,897 km/h. At that speed, how many days would it take to travel 54.6 million km, the minimum distance from Earth to Mars? Express your answer to the nearest whole number.

173. _____ Yeong multiplies two-digit positive integers AB and CD. If the digits A, B, C and D are all distinct, what is the greatest possible value of the product?

174. $ _____ Karli paid $3.00 for lunch every day she attended school. During a six-week period, Karli attended school every Monday through Friday, with the exception of one school holiday. What is the total amount that Karli spent on school lunches during this six-week period?

175. _____ What is the sum of the mean, median, mode and range of the numbers 15, 33, 24, 10, 20 and 24?

176. _____ gallons A certain race car consumes 1.3 gallons of fuel during each lap of a race. If each lap is 2.5 miles, and the entire race is 500 miles, how much fuel does the race car consume from start to finish?



177. _____ inches One television screen measures 56 inches long and 33 inches wide. A smaller, geometrically similar television screen measures 48 inches long. What is the width of the smaller screen? Express your answer as a decimal to the nearest tenth.

178. _____ mi/h  Lindsay starts at the peak of a mountain, and it takes her 50 minutes to hike 15,000 feet. What was her average walking speed, in miles per hour, given 1 mile = 5280 feet? Express your answer as a decimal to the nearest tenth.

179. _____ Let a be the arithmetic mean of 3.27 and 17.95. Let b be the product of 32.7 and 0.4382. Let c be the quotient of 2.637 and 0.316. Let d be the absolute difference between 793.241 and 804.3692. What is the numerical value of the median of a, b, c and d? Express your answer as a decimal to the nearest hundredth.

180. _____ units^2 Jessie makes butterfly wings by shading four sections of a regular hexagon as shown. If the hexagon has side length $4\sqrt{3}$ units, what is the area of the shaded region? Express your answer in simplest radical form.

Workout 2

181. _____ units^2 What is the area of a triangle with sides of length 16, 30 and 34?

182. _____ gallons How many gallons are in the volume of a cylinder with height 12 inches and diameter 14 inches if 1 gallon = 231 in^3? Express your answer to the nearest whole number.

183. _____ If $x \mathbin{@} y = \text{LCM}(x, y)$ and $x \mathbin{\#} y = \text{GCD}(x, y)$, what is the value of $((2 \mathbin{@} 7)^2 \mathbin{\#} 42) \mathbin{\#} 105$?

184. _____ minutes On a digital clock that shows hours and minutes, for how many minutes, during a single day, between 4:00 a.m. and 6:00 a.m. will the sum of the digits be 10?

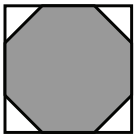
185. _____ cm^3 The density of an object is the mass of the object per unit volume. The density of water is 1 gram/cm^3, and the density of ice is 91.67% that of water. If 1 kilogram of water completely freezes, what is the volume, in cubic centimeters, of the resulting ice block? Express your answer to the nearest whole number.

186. _____ If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = aek + bfg + cdh - ceg - afh - bdk$, what is the value of $\begin{vmatrix} 2 & -5 & 3 \\ 0 & 4 & -6 \\ -1 & 8 & 7 \end{vmatrix}$?

187. _____ degrees A certain trapezoid has these properties: its diagonals are congruent and perpendicular to each other, and its longer base length is equal to the length of a diagonal. What is the sum of the degree measures of the two acute angles of this trapezoid?

188. _____ sub-sets How many subsets of {1, 2, 3, 4, 6, 8, 10, 15} are there for which the sum of the elements is 15?

189. _____ % The figure shows a shaded regular octagon inscribed in a square. What percent of the figure is shaded? Express your answer to the nearest whole number.



190. _____ minutes

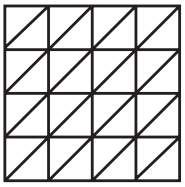
Spike can dig 8 holes in 3 hours. Butch can dig 7 holes in 4 hours. Lucky can dig 6 holes in 5 hours. How many minutes will it take them to dig 3 holes if all three work together? Express your answer to the nearest whole number.

Workout 3

191. _____ Let q be the sum of the lengths of all 12 edges of a cube, and let A be the total surface area of the cube. If $q \times A$ is k times the volume of the cube, what is the value of k?

192. _____ Alta's favorite sphere has six times as much surface area compared to her favorite cylinder, which is right circular. If the height of her favorite cylinder is equal to its diameter, what is the ratio of the volume of Alta's favorite cylinder to the volume of her favorite sphere? Express your answer as a common fraction.

193. _____ triangles How many triangles of any size are in the figure shown here?



194. _____ page Rex reads a novel at a constant rate while Wren writes a report at a constant rate. Rex finishes page 313 of his novel at the same time Wren finishes page 5 of her report. Rex later finishes page 409 of his novel when Wren finishes page 9 of her report. What page of his novel will Rex finish when Wren finishes page 12 of her report?

195. _____ The Fibonacci sequence is defined by the function $F(n) = F(n - 2) + F(n - 1)$, for $n > 2$ and $F(1) = F(2) = 1$. What is the value of $F(11)$?

196. _____ What is the ratio of the area of the largest square that can fit inside a circle of diameter 2 cm compared to the area of the largest square that can fit inside a semicircle of diameter 2 cm? Express your answer as a common fraction.

197. _____ If $20! = 2^a \times b$ for some integer b, what is the greatest possible value of a?

198. _____ % At a certain middle school, the 6th-grade class of 390 increased by 10%, the 7th-grade class of 350 increased by 22% and the 8th-grade class of 420 increased by 20%. Overall, what was the percentage increase of students for this middle school? Express your answer to the nearest tenth.

199. _____ units^3 A 6-8-10 right triangle is rotated about the side of length 10 units. What is the volume of the resulting solid? Express your answer as a decimal to the nearest tenth.

200. _____ 

Vikram attempted to calculate 67 × 58, but he entered exactly one digit incorrectly, resulting in a product of 3596. What is the sum of the two values he actually multiplied?

Workout 4

201. $\_\_\_\_\_$ Alice's car travels 29 miles per gallon of gas. Phil's car uses 0.02 gallon of gas per mile traveled. If gas costs $2.50 per gallon, what is the absolute difference in the amounts Alice and Phil would each spend on the gas needed to drive 113 miles?

202. _____ A cheese cube is sliced diagonally from one edge to another, as shown. One piece of the cheese is melted and then reformed into a perfect sphere. What is the ratio of the side length of the original cheese cube to the radius of the cheese sphere? Express your answer as a decimal to the nearest hundredth.

203. ____ : ____ a.m. Three consecutive rounds of an X-game last 24 minutes, 48 minutes and 96 minutes. If the game begins at 6:00 a.m., at what time will the third round end?

204. _____ meters

1.2	1 3 6 6
1.3	2 4 5 5 7 9
1.4	0 1 4 8
1.5	0 3 4 6 8 8 9
1.6	0 5 7
1.7	1

Ms. Santa asks the 25 students in her math class to measure their heights in meters. The students' heights are recorded in the stem-and-leaf plot shown here, where 1.2 | 1 represents 1.21 meters. What is the median height of the students in Ms. Santa's class? Express your answer as a decimal to the nearest hundredth.

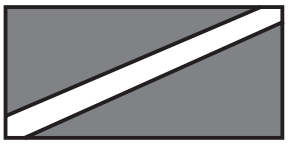
205. _____ ordered pairs How many ordered pairs of positive integers (a, b) satisfy the equation $\frac{2}{a} - \frac{3}{b} = \frac{1}{5}$?

206. _____ steps 

R.J.'s pedometer indicates that he has walked 10,002 steps and equates that to traveling 4.11 miles. Based on this, how many additional steps must he walk to travel the equivalent of 5 miles total? Express your answer to the nearest whole number.

207. $\_\_\_\_\_$ A liter of fuel cost £1.33. At that rate, what would be the cost, in U.S. currency, of 1 gallon of fuel, given that 1 gallon ≈ 3.78541 liters and US$1.30 ≈ £1.00?

208. $\_\_\_\_\_$ A 10-meter by 20-meter garden is going to have a concrete walkway constructed diagonally through the middle as shown. The longest sides of the walkway each start 1 meter from the closest corner of the garden. If the walkway will have a uniform depth of 10 cm and the concrete costs $70 per cubic meter, what will be the cost of the concrete used for the walkway?



209. _____ years old On his 15th birthday, Bo invests $10,000 in a bank that pays 7.5% interest, compounded annually. At this rate and assuming Bo makes no deposits or withdrawals, how old will Bo be on the first birthday when his account balance is at least $30,000?

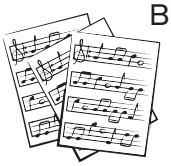
210. _____ permu-tations How many different two-letter permutations can be made from two distinct letters in the word MATHCOUNTS?

Workout 5

211. _____ cm A regular octagon with side length 4 cm is concentric with a circle, as shown. If the area of the circle is equal to the area of the shaded region between the shapes, what is the radius of the circle? Express your answer as a decimal to the nearest tenth.

212. _____ ounces Jonas has a $1\frac{1}{2}$-inch by 4-inch by 3-inch rectangular block of tofu weighing 16 ounces. He cuts the block into smaller pieces, each of which is a triangular prism having a right triangle base with legs of length $1\frac{1}{3}$ inch and $\frac{3}{4}$ inch. If each tofu prism has height $\frac{1}{2}$ inch, what is the weight of one tofu prism? Express your answer as a common fraction.

213. _____ sequences



Blake has a repertoire of 15 songs, of which 5 are acoustic. In his concert, he plans to perform 10 distinct songs, of which 4 will be acoustic. He wants to perform the 4 acoustic songs consecutively in the second half of the concert but doesn't want to end the concert with an acoustic song. How many different sequences of songs can Blake perform?

214. _____ % Each time Chris bats, he has a 37% probability of getting a hit, independent of his previous performance at bat. What is his percent probability of getting at least two hits in four times at bat? Express your answer to the nearest whole number.

215. _____ % The Bobcats had a win-loss record of 41-23 before their star player, Melissa, got injured. Melissa didn't play in any more games, and the Bobcats finished the season with an overall win-loss record of 54-34. What is the absolute difference in the Bobcats' win percentages with and without Melissa? Express your answer to the nearest whole number.

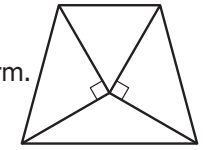
216. _____ points On a test, Walter scored 86, Bev scored 72, Claire scored 61 and Rachel scored 93. Tyler didn't remember his score, but he knew that his score was both the mean and the median of all five students' scores. What was Tyler's score?

217. _____ The two values of x that satisfy the equation $x^2 + bx + c = 0$, for integers b and c, are $x = 6$ and $x = -4$. What is the value of b?

218. _____ An eight-digit number N has one 2, three 3s, two 4s and two 5s as digits. If N can be expressed as a power of 2, what is the value of N?

219. $ _____ At Randolf's furniture store, a standard breakfast table has a price of $1000. Karthik wants to purchase the luxury version of the table, which costs 35% more than the standard one, and he has a coupon that gives him a 15% discount on his purchase. After applying 6% sales tax to the discounted price, how much will Karthik pay for the luxury table?

220. _____ Tarin and Sonja construct the trapezoid shown, which contains an equilateral triangle and three isosceles triangles, two of which are right triangles. What is the ratio of the length of the shorter base to the length of the longer base of the trapezoid? Express your answer as a common fraction in simplest radical form.



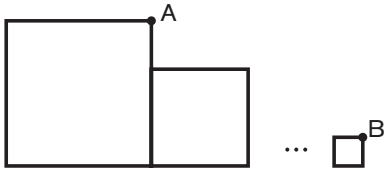
Workout 6

221. _____ spells

Ronald is attempting to cast a levitation spell. On his first attempt, he has a 20% chance of success. Ronald's likelihood of success improves by 5% after each failed attempt. For example, his second attempt has a 25% chance of success if he fails on the first attempt. What is the minimum number of spells that he must attempt to guarantee a successful casting?

222. _____ units

In a sequence of adjacent squares, a portion of which is shown, the side length of each successive square is two-thirds that of the preceding square. In the completed sequence, the areas of the largest and smallest squares are 6561 units^2 and 256 units^2, respectively. What is the length of the segment with endpoints A and B? Express your answer to the nearest whole number.



223. _____ ways

A cheerleading squad consists of ten cheerleaders of ten different heights. How many ways are there for the cheerleaders to line up for a photo in two rows of five people each so that each cheerleader in the back row is taller than the one immediately in front?

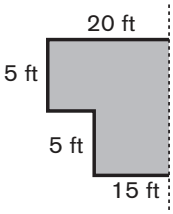
224. $ _____

The math club purchased a number of gadgets to sell. Each box of gadgets cost the club $75 and contained 30 gadgets. In the first 15 days, the club members sold 22 gadgets each day at the regular price of $6.80 each. In the next 10 days, they sold 17 gadgets per day at a discount of 25% off. They then sold the remaining 220 gadgets at 30% off the discounted price. After all the gadgets were sold, what was the math club's total profit?

225. _____

If A, B and C are the digits of the base-five number ABC, which is equal to the base-ten number 47, what digit does B represent?

226. _____ feet



The figure, which shows half of the cross section of a pool, can be rotated 360 degrees around the dotted line to create a three-dimensional pool with depth 10 feet in the center and 5 feet at the outer edge. What is the average depth of the pool? Express your answer as a decimal to the nearest tenth.

227. _____ games

In his first 319 career games, goaltender Braden Holtby had 200 wins and 119 losses. If he were to maintain this rate of winning games, what is the least number of games he would need to play in his career to pass Martin Brodeur's career record of 691 wins?

228. _____

What is the least positive odd integer with exactly nine natural number divisors?

229. _____ seconds

Noa completes $5\frac{1}{3}$ laps around a track in the same amount of time it takes Lev to complete 6 laps. Lev is running at a pace of 7.5 minutes per mile. How many seconds longer will it take Noa to run 1 mile, compared to Lev? Express your answer to the nearest whole number.

230. _____ in^2

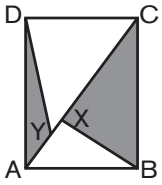
What is the area of a triangle with side lengths of 4, 6 and 8 inches? Express your answer in simplest radical form.

Workout 7

231. _____ If a number x plus its reciprocal is equal to 4, what is the absolute difference between x and its reciprocal? Express your answer in simplest radical form.

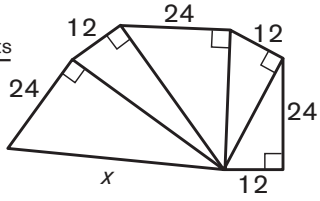
232. $ _____ For making house numbers, 5-inch digits can be purchased online for $8.79 each for the digits 1, 3, 5 and 6 and $5.98 each for the digits 0, 2, 4, 7, 8 and 9. Ashera wishes to create consecutive house numbers beginning with 5100 and ending with 5200. What is the total cost to purchase the necessary digits online, excluding tax and shipping?

233. _____ units^2 Rectangle ABCD is shown here with AB = 6 and BC = 8. If X and Y lie on diagonal AC, and XY = 1, what is the total area of shaded triangles BCX and DAY? Express your answer as a decimal to the nearest tenth.



234. _____ orders  Fran has four dogs whose ages are 3, 4, 5 and 6 years. She feeds them one at a time according to the rule that a younger dog can never eat right before a dog that is only one year older. In how many different orders can Fran feed her four dogs?

235. _____ What is the arithmetic mean of the terms of a five-term geometric sequence of positive numbers whose second term is 15 and whose fourth term is 135?

236. _____ units 

Five right triangles are constructed adjacent to each other as shown. All five triangles share a vertex. The longer leg of each triangle, except the rightmost triangle, is the hypotenuse of an adjacent triangle. What is the length of the longest hypotenuse, labeled x? Express your answer in simplest radical form.

237. _____ cm An equilateral triangle and a regular hexagon have a combined perimeter of 42 cm and a combined area of $49\sqrt{3}$ cm^2. What is the side length of the hexagon? Express your answer as a decimal to the nearest tenth.

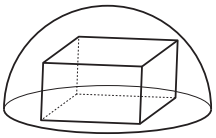
238. _____ Ty flips a fair coin 20 times. What is the probability that it lands heads up and tails up equal numbers of times? Express your answer as a decimal to the nearest thousandth.

239. _____ troy ounces One hundred years ago, a newly minted Walking Liberty Half Dollar contained 0.36 troy ounce of silver, a Standing Liberty Quarter contained 90% silver (by mass) and a Mercury Dime was made from the same alloy as the other two coins. If the mass of each coin was proportional to its face value, what was the total mass of a Mercury Dime? Express your answer as a decimal to the nearest hundredth.

240. _____ units Two right triangles have areas in the ratio 1:2. If the smaller triangle has legs of length 3 units and 4 units, and the larger triangle has a side of length 8 units, what is the sum of the possible lengths of the hypotenuse of the larger triangle? Express your answer in simplest radical form.

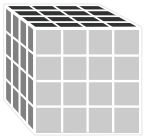
Workout 8

241. _____ inches Scott thinks that Miguel is 6 feet 3 inches tall. If Miguel's height is actually 96% of Scott's estimate, how many inches tall is Miguel?

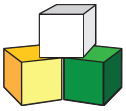
242. _____ in^3  A hemisphere of radius 6 inches lies flat on its circular base, and a rectangular prism with two square bases lies inside the hemisphere, with one square base on the base of the hemisphere and all four vertices of the other square base touching the upper boundary of the hemisphere. The height of the prism is one-third of the length of the square base. What is the volume of the prism? Express your answer as a decimal to the nearest tenth.

243. _____ If N is a perfect square and a divisor of 13!, what is the greatest possible value of N?

244. _____ lines When 64 unit cubes are glued together seamlessly to form a 4 × 4 × 4 cube, as shown, there are 76 distinct lines that pass through the centers of exactly four unit cubes. If 1000 unit cubes are glued together seamlessly to form a 10 × 10 × 10 cube, how many distinct lines pass through the centers of ten unit cubes?

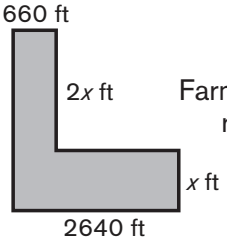


245. _____ Suppose (a, b, c) is an ordered triple such that $a^2 + b^2 = c^2$. If $b = x + y\sqrt{2}$ when $a = 12 + 10\sqrt{2}$ and $c = 15 + 8\sqrt{2}$, what is the positive integer value of $x + y$?

246. _____ yellow blocks  Camsie has a large number of identical yellow blocks, identical green blocks and identical white blocks. Using a balance scale, she finds that 3 green blocks will balance 1 yellow plus 3 white blocks and that 5 white blocks will balance 1 green plus 5 yellow blocks. How many yellow blocks will balance 30 green plus 60 white blocks?

247. _____ A box contains only quarters and dimes. If there were 10% more quarters, there would be 7.5% more money in the box. What is the ratio of the original number of quarters to the original number of dimes in the box? Express your answer as a common fraction.

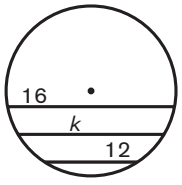
248. _____ feet



Farmer John's field has area 60 acres. The field is in the shape of an "L", with right angles at every vertex and dimensions as shown. If one acre is equal in area to a strip of land 40 rods long and 4 rods wide, and one rod is equal in length to 16.5 feet, what is value of x in feet?

249. _____ ways Notice that 2019 can be written as the sum of consecutive integers in increasing order in numerous ways, for example 1009 +1010 = 2019 and (−2018) + (−2017) + ... + 2018 + 2019 = 2019. Including these two examples, in how many ways can 2019 be written as the sum of at least two consecutive integers in increasing order?

250. _____ units Three parallel chords of lengths 12, k and 16 units are in a circle, as shown. If the chord of length k is equally spaced 2 units away from each of the other chords, what is the value of k? Express your answer in simplest radical form.



MATHCOUNTS® COMPETITION SERIES EST. 1983

OFFICIAL RULES + PROCEDURES

The following rules and procedures govern all MATHCOUNTS competitions. The MATHCOUNTS Foundation reserves the right to alter these rules and procedures at any time. **Coaches are responsible for being familiar with the rules and procedures outlined in this handbook.** Coaches should bring any difficulty in procedures or in student conduct to the immediate attention of the appropriate chapter, state or national official. Students violating any rules may be subject to immediate disqualification.

Any questions regarding the MATHCOUNTS Competition Series Official Rules + Procedures articulated in this handbook should be addressed to the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.

REGISTRATION

The fastest and easiest way to register for the MATHCOUNTS Competition Series is online at www.mathcounts.org/compreg.

For your school to participate in the MATHCOUNTS Competition Series, a school representative is required to complete a registration form and pay the registration fees. A school representative can be a teacher, administrator or parent volunteer who has received expressed permission from his/her child's school administration to register. By completing the Competition Series Registration Form, the coach attests to the school administration's permission to register students for MATHCOUNTS.

School representatives can register online at www.mathcounts.org/compreg or download the Competition Series Registration Form and mail or email a scanned copy of it to the MATHCOUNTS national office. Refer to the Critical 2019-2019 Dates on pg. 10 of this handbook for contact information.

WHAT REGISTRATION COVERS: Registration in the Competition Series entitles a school to:

1) send 1-10 student(s)—depending on number registered—to the Chapter Competition. *Students can advance beyond the chapter level, but this is determined by their performance at the competition.*
2) receive the School Competition Kit, which includes the *2018-2019 MATHCOUNTS School Handbook*, one recognition ribbon for each registered student, 10 student participation certificates and a catalog of additional coaching materials. *Mailings of School Competition Kits will occur on a rolling basis through December 31, 2018.*
3) receive online access to the 2019 School Competition, along with electronic versions of other competition materials at www.mathcounts.org/coaches. *Coaches will receive an email notification no later than November 2, 2018 when the 2019 School Competition is available online.*

Your state or chapter coordinator will be notified of your registration, and then you will be informed of the date and location of your Chapter Competition. **If you have not been contacted by mid-January with competition details, it is your responsibility to contact your local coordinator** to confirm that your registration has been properly routed and that your school's participation is expected. Coordinator contact information is available at www.mathcounts.org/findmycoordinator.

DEADLINES: The sooner your Registration Form is received, the sooner you will receive your preparation materials. To guarantee your school's participation, submit your registration by one of the following deadlines:

Early Bird Discount Deadline: November 2, 2018	Online registrations: submitted by 11:59 PST Emailed forms: received by 11:59 PST Mailed forms: postmarked by November 2, 2018
Regular Registration Deadline: December 14, 2018*	Online registrations: submitted by 11:59 PST Emailed forms: received by 11:59 PST Mailed forms: postmarked by December 14, 2018

*Late Registrations may be accepted at the discretion of the MATHCOUNTS national office and your local coordinators, but are not guaranteed. If a school's late registration is accepted, an additional $20 processing fee will be assessed.

REGISTRATION FEES: The cost of your school's registration depends on when your registration is postmarked/emailed/submitted online. The cost of your school's registration covers the students for the entire Competition Series; there are no additional registration fees to compete at the state or national level. Title I schools (as affirmed by a school's administration) receive a 50% discount off the total cost of their registration.

Early Bird Registrations (by November 2, 2018)	**$30 per student** $120 for 1 team of 4 | $300 for 1 team of 4 + 6 individuals
Regular Registrations (by December 14, 2018)	**$35 per student** $140 for 1 team of 4 | $350 for 1 team of 4 + 6 individuals
Late Registrations (after December 14, 2018)	**$35 per student + $20 late fee on entire order** $160 for 1 team of 4 | $370 for 1 team of 4 + 6 individuals

CANCELLATION FEES: Registered schools that need to cancel their Competition Series registration must notify the MATHCOUNTS national office in writing via email or mail. Schools may request and receive a full refund minus a $30 non-refundable cancellation fee to cover refund processing and the cost of materials shipped to the school. MATHCOUNTS will verify a school's non-participation with local coordinators and reserves the right to refuse a refund request. No cancellations or refund requests will be processed after February 1, 2019. *This fee does not apply to schools that reduce their number of registered students but remain registered with at least one student.*

ELIGIBILITY REQUIREMENTS

Eligibility requirements for the MATHCOUNTS Competition Series are different from other MATHCOUNTS programs. Eligibility for the National Math Club or the Math Video Challenge does not guarantee eligibility for the Competition Series.

WHO IS ELIGIBLE:

- U.S. students enrolled in the 6th, 7th or 8th grade can participate in MATHCOUNTS competitions.
- Schools that are the students' official school of record can register.
- Any type of school, of any size, can register—public, private, religious, charter, virtual or homeschools—but virtual and homeschools must fill out additional forms to participate (see pgs. 39-40).
- Schools in 50 U.S. states, District of Columbia, Guam, Puerto Rico and Virgin Islands can register.
- Overseas schools that are affiliated with the U.S. Departments of Defense and State can register.

WHO IS NOT ELIGIBLE:

- Students who are not full-time 6th, 7th or 8th graders cannot participate, even if they are taking middle school math classes.
- Academic centers, tutoring centers or enrichment programs that do not function as students' official school of record cannot register. *If it is unclear whether your educational institution is considered a school, please contact your local Department of Education for specific criteria governing your state.*
- Schools located outside of the U.S. states and territories listed on the previous page cannot register.
- Overseas schools not affiliated with the U.S. Departments of Defense or State cannot register.

NUMBER OF STUDENTS ALLOWED: A school can register a maximum of one team of four students and six individuals; these 1-10 student(s) will represent the school at the Chapter Competition. Any number of students can participate at the school level. Prior to the Chapter Competition, coaches must notify their chapter coordinator of which students will be team members and which students will compete as individuals.

NUMBER OF YEARS ALLOWED: Participation in MATHCOUNTS competitions is limited to 3 years for each student, but there is no limit to the number of years a student may participate in school-based coaching.

WHAT TEAM REGISTRATION MEANS: Members of a school team will participate in the Target, Sprint and Team Rounds. Members of a school team also will be eligible to qualify for the Countdown Round (where conducted). Team members will be eligible for team awards, individual awards and progression to the state and national levels based on their individual and/or team performance. It is recommended that your strongest four Mathletes form your school team. Teams of fewer than four will be allowed to compete; however, the team score will be computed by dividing the sum of the team members' scores by four (see pg. 43), meaning, teams of fewer than four students will be at a disadvantage. *Only one team (of up to four students) per school is eligible to compete.*

WHAT INDIVIDUAL REGISTRATION MEANS: Students registered as individuals will participate in the Target and Sprint Rounds, but not the Team Round. Individuals will be eligible to qualify for the Countdown Round (where conducted). Individuals also will be eligible for individual awards and progression to the state and national levels. A student registered as an "individual" may not help his/her school's team advance to the next level of competition. *Up to six students may be registered in addition to or in lieu of a school team.*

HOW STUDENTS ENROLLED PART-TIME AT TWO SCHOOLS PARTICIPATE: *A student may compete only for his/her official school of record.* A student's school of record is the student's base or main school. A student taking limited course work at a second school or educational center may not register or compete for that second school or center, even if the student is not competing for his/her school of record. MATHCOUNTS registration is not determined by where a student takes his or her math course. If there is any doubt about a student's school of record, the chapter or state coordinator must be contacted for a decision before registering.

HOW SMALL SCHOOLS PARTICIPATE: MATHCOUNTS does not distinguish between the sizes of schools for Competition Series registration and competition purposes. Every "brick-and-mortar" school will have the same registration allowance of up to one team of four students and/or up to six individuals. A school's participants may not combine with any other school's participants to form a team when registering or competing.

HOW HOMESCHOOLS PARTICIPATE: Homeschools and/or homeschool groups in compliance with the homeschool laws of the state in which they are located are eligible to participate in MATHCOUNTS competitions in accordance with all other rules. Homeschool coaches must complete the 2018-2019 Homeschool + Virtual School Participation Form, verifying that students from the homeschool or homeschool group are in the 6th, 7th or 8th grade and that each homeschool complies with applicable state laws. Forms can be downloaded at www.mathcounts.org/competition and must be submitted to the MATHCOUNTS national office in order for registrations to be processed.

HOW VIRTUAL SCHOOLS PARTICIPATE: Virtual schools that want to register must contact the MATHCOUNTS national office by December 7, 2018 for specific registration details. Any student registering as a virtual school student must compete in the MATHCOUNTS Chapter Competition assigned according to the student's home address. Additionally, virtual school coaches must complete the 2018-2019 Homeschool + Virtual School Participation Form, verifying that the students from the virtual school are in the 6th, 7th or 8th grade and that the virtual school complies with applicable state laws. Forms must be submitted to the national office in order for registrations to be processed; forms can be downloaded at www.mathcounts.org/competition.

WHAT IS DONE FOR SUBSTITUTIONS OF STUDENTS: Coaches determine which students will represent the school at the Chapter Competition. Coaches cannot substitute team members for the State Competition unless a student voluntarily releases his/her position on the school team. Additional requirements and documentation for substitutions (such as requiring parental release or requiring the substitution request be submitted in writing) are at the discretion of the State Coordinator. A student being added to a team need not be a student who was registered for the Chapter Competition as an individual. Coaches cannot make substitutions for students progressing to the State Competition as individuals. At all levels of competition, student substitutions are not permitted after on-site competition registration has been completed.

WHAT IS DONE FOR RELIGIOUS OBSERVANCES: A student who is unable to attend a competition due to religious observances may take the written portion of the competition up to one week in advance of the scheduled competition. In addition, all competitors from that student's school must take the Sprint and Target Rounds at the same earlier time. If the student who is unable to attend the competition due to a religious observance: (1) is a member of the school team, then the team must take the Team Round at the same earlier time; (2) is not part of the school team, then the team has the option of taking the Team Round during this advance testing or on the regularly scheduled day of the competition with the other school teams. The coordinator must be made aware of the team's decision before the advance testing takes place. *Advance testing will be done at the discretion of the chapter and state coordinators. If advance testing is deemed possible, it will be conducted under proctored conditions.* Students who qualify for an official Countdown Round but are unable to attend will automatically forfeit one place standing.

WHAT IS DONE FOR STUDENTS WITH SPECIAL NEEDS: Reasonable accommodations may be made to allow students with special needs to participate. However, many accommodations that are employed in a classroom or teaching environment cannot be implemented in the competition setting. Accommodations that are not permissible include, but are not limited to: granting a student extra time during any of the competition rounds or allowing a student to use a calculator for the Sprint or Countdown Rounds. *A request for accommodation of special needs must be directed to chapter or state coordinators in writing at least three weeks in advance of the Chapter or State Competition.* This written request should thoroughly explain a student's special need, as well as what the desired accommodation would entail. In conjunction with the MATHCOUNTS Foundation, coordinators will review the needs of the student and determine if any accommodations will be made. In making final determinations, the feasibility of accommodating these needs at the National Competition will be taken into consideration.

LEVELS OF COMPETITION

There are four levels in the MATHCOUNTS Competition Series: school, chapter (local), state and national. Competition questions are written for 6th, 7th and 8th graders. The competitions can be quite challenging, particularly for students who have not been coached using MATHCOUNTS materials. All competition materials are prepared by the national office.

SCHOOL COMPETITIONS (TYPICALLY HELD IN JANUARY 2019): After several months of coaching, schools registered for the Competition Series should administer the 2019 School Competition to all interested

students. The School Competition should be an aid to the coach in determining competitors for the Chapter Competition. *Selection of team and individual competitors is entirely at the discretion of the coach and does not need to be based solely on School Competition scores.* School Competition materials are sent to the coach of a school, and it may be used by the teachers and students only in association with that school's programs and activities. The current year's School Competition questions must remain confidential and may not be used in outside activities, such as tutoring sessions or enrichment programs with students from other schools. For updates or edits, please check www.mathcounts.org/coaches before administering the School Competition.

It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better math skills in all students, not just in those students who will be competing. Therefore, it is suggested that the coach postpone selection of competitors until just prior to the Chapter Competition.

CHAPTER COMPETITIONS (HELD FEB. 1–28, 2019): The Chapter Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be conducted. The chapter and state coordinators determine the date and location of the Chapter Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The winning team will advance to the State Competition. Additionally, the two highest-ranking competitors not on the winning team (who may be registered as individuals or as members of a team) will advance to the State Competition. This is a minimum of six advancing Mathletes (assuming the winning team has four members). Additional teams and/or individuals also may progress at the discretion of the state coordinator, but the policy for progression must be consistent for all chapters within a state.

STATE COMPETITIONS (HELD MAR. 1–31, 2019): The State Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be included. The state coordinator determines the date and location of the State Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The four highest-ranked Mathletes and the coach of the winning team from each State Competition will receive an all-expenses-paid trip to the National Competition.

2019 RAYTHEON MATHCOUNTS NATIONAL COMPETITION (HELD MAY 12–13 IN ORLANDO, FL): The National Competition consists of the Sprint, Target, Team and Countdown Rounds (conducted officially). Expenses of the state team and coach to travel to the National Competition will be paid by MATHCOUNTS. The national program does not make provisions for the attendance of additional students or coaches. All national competitors will receive a plaque and other items in recognition of their achievements. Winning teams and individuals also will receive medals, trophies and college scholarships.

COMPETITION COMPONENTS

The four rounds of a MATHCOUNTS competition, each described below, are designed to be completed in approximately three hours:

TARGET ROUND (approximately 30 minutes): In this round eight problems are presented to competitors in four pairs (six minutes per pair). The multi-step problems featured in this round engage Mathletes in mathematical reasoning and problem-solving processes. *Problems assume the use of calculators.*

SPRINT ROUND (40 minutes): Consisting of 30 problems, this round tests accuracy, with the time period allowing only the most capable students to complete all of the problems. *Calculators are not permitted.*

TEAM ROUND (20 minutes): In this round, interaction among team members is permitted and encouraged as they work together to solve 10 problems. *Problems assume the use of calculators.*

Note: The order in which the written rounds (Target, Sprint and Team) are administered is at the discretion of the competition coordinator.

COUNTDOWN ROUND: A fast-paced oral competition for top-scoring individuals (based on scores on the Target and Sprint Rounds), this round allows pairs of Mathletes to compete against each other and the clock to solve problems. *Calculators are not permitted.*

At Chapter and State Competitions, a Countdown Round (1) may be conducted officially, (2) may be conducted unofficially (for fun) or (3) may be omitted. However, the use of an official Countdown Round must be consistent for all chapters within a state. In other words, *all* chapters within a state must use the round officially in order for *any* chapter within a state to use it officially. All students, whether registered as part of a school team or as individual competitors, are eligible to qualify for the Countdown Round.

An official Countdown Round determines an individual's final overall rank in the competition. If a Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed, as described below.*

- The top 25% of students, up to a maximum of 10, are selected to compete. These students are chosen based on their Individual Scores.
- The two lowest-ranked students are paired; a question is read and projected, and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if s/he answers correctly, a point is scored. If a student answers incorrectly, the other student has the remainder of the 45 seconds to answer.
- Three total questions are read to the pair of students, one question at a time, and the student who scores the higher number of points (not necessarily 2 out of 3) progresses to the next round and challenges the next-higher-ranked student.
- If students are tied in their matchup after three questions (at 1-1 or 0-0), questions should continue to be read until one is successfully answered. The first student who answers an additional question correctly progresses to the next round.
- This procedure continues until the 4th-ranked Mathlete and his/her opponent compete. For the final four matchups, the first student to correctly answer three questions advances.
- The Countdown Round proceeds until a 1st place individual is identified. More details about Countdown Round procedures are included in the 2019 School Competition.

**Rules for the Countdown Round change for the National Competition.*

An unofficial Countdown Round does not determine an individual's final overall rank in the competition, but is done for practice or for fun. The official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners solely on the basis of students' scores in the Target and Sprint Rounds of the competition.

SCORING

MATHCOUNTS Competition Series scores do not conform to traditional grading scales. Coaches and students should view an Individual Score of 23 (out of a possible 46) as highly commendable.

INDIVIDUAL SCORE: calculated by taking the sum of the number of Sprint Round questions answered correctly and twice the number of Target Round questions answered correctly. There are 30 questions in the Sprint Round and eight questions in the Target Round, so the maximum possible Individual Score is $30 + 2(8) = 46$. If used officially, the Countdown Round yields final individual standings.

TEAM SCORE: calculated by dividing the sum of the team members' Individual Scores by four (even if the team has fewer than four members) and adding twice the number of Team Round questions answered correctly. The highest possible Individual Score is 46. Four students may compete on a team, and there are 10 questions in the Team Round. Therefore, the maximum possible Team Score is ((46 + 46 + 46 + 46) ÷ 4) + 2(10) = 66.

TIEBREAKING ALGORITHM: used to determine team and individual ranks and to determine which individuals qualify for the Countdown Round. In general, questions in the Target, Sprint and Team Rounds increase in difficulty so that the most difficult questions occur near the end of each round. In a comparison of questions to break ties, generally those who correctly answer the more difficult questions receive the higher rank. The guidelines provided below are very general; competition officials receive more detailed procedures.

- *Ties between individuals:* the student with the higher Sprint Round score will receive the higher rank. If a tie remains after this comparison, specific groups of questions from the Target and Sprint Rounds are compared.
- *Ties between teams:* the team with the higher Team Round score, and then the higher sum of the team members' Sprint Round scores, receives the higher rank. If a tie remains after these comparisons, specific questions from the Team Round will be compared.

RESULTS DISTRIBUTION

Coaches should expect to receive the scores of their students, as well as a list of the top 25% of students and top 40% of teams, from their competition coordinators. In addition, single copies of the blank competition materials and answer keys may be distributed to coaches after all competitions at that level nationwide have been completed. *Before distributing blank competition materials and answer keys, coordinators must wait for verification from the national office that all such competitions have been completed.* Both the problems and answers from Chapter and State Competitions will be posted on the MATHCOUNTS website following the completion of all competitions at that level nationwide, replacing the previous year's posted tests.

Student competition papers and answers will not be viewed by or distributed to coaches, parents, students or other individuals. Students' competition papers become the confidential property of MATHCOUNTS.

ADDITIONAL RULES

All answers must be legible.

Pencils and paper will be provided for Mathletes by competition organizers. However, students may bring their own pencils, pens and erasers if they wish. They may not use their own scratch paper or graph paper.

Use of notes or other reference materials (including dictionaries and translation dictionaries) is prohibited.

Specific instructions stated in a given problem take precedence over any general rule or procedure.

Communication with coaches is prohibited during rounds but is permitted during breaks. All communication between guests and Mathletes is prohibited during competition rounds. Communication between teammates is permitted only during the Team Round.

Calculators are not permitted in the Sprint and Countdown Rounds, but they are permitted in the Target, Team and Tiebreaker (if needed) Rounds. When calculators are permitted, students may use any calculator (including programmable and graphing calculators) that does not contain a QWERTY (typewriter-like) keypad. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Smart phones, laptops, tablets, iPods®, personal

digital assistants (PDAs) and any other "smart" devices are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.

Coaches are responsible for ensuring their students use acceptable calculators, and students are responsible for providing their own calculators. Coordinators are not responsible for providing Mathletes with calculators or batteries before or during MATHCOUNTS competitions. Coaches are strongly advised to bring backup calculators and spare batteries to the competition for their team members in case of a malfunctioning calculator or weak or dead batteries. Neither the MATHCOUNTS Foundation nor coordinators shall be responsible for the consequences of a calculator's malfunctioning.

Pagers, cell phones, tablets, iPods® and other MP3 players should not be brought into the competition room. Failure to comply could result in dismissal from the competition.

Should there be a rule violation or suspicion of irregularities, the MATHCOUNTS coordinator or competition official has the obligation and authority to exercise his/her judgment regarding the situation and take appropriate action, which might include disqualification of the suspected student(s) from the competition.

FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they both may be omitted. Answers in the form ($) *a.bc* should be rounded to the nearest cent, unless otherwise specified. Examples:
Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:
Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3
Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

Competition Coach Toolkit

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection useful.

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}$
$\frac{1}{4}$	0.25	25
$\frac{1}{5}$	0.2	20
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$
$\frac{1}{8}$	0.125	12.5
$\frac{1}{9}$	$0.\overline{1}$	$11.\overline{1}$
$\frac{1}{10}$	0.1	10
$\frac{1}{11}$	$0.\overline{09}$	$9.\overline{09}$
$\frac{1}{12}$	$0.08\overline{3}$	$8.\overline{3}$

Common Arithmetic Series

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 3 + 5 + 7 + \cdots + (2n-1) = n^2$$

$$2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$$

Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n-r)!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

Divisibility Rules

2: units digit is 0, 2, 4, 6 or 8

3: sum of digits is divisible by 3

4: two-digit number formed by tens and units digits is divisible by 4

5: units digit is 0 or 5

6: number is divisible by both 2 and 3

8: three-digit number formed by hundreds, tens and units digits is divisible by 8

9: sum of digits is divisible by 9

10: units digit is 0

Equation of a Line

Standard Form

$Ax + By = C$

Slope-Intercept Form

$y = mx + b$

m = slope $\qquad$ b = y-intercept

Point-Slope Form

$y - y_1 = m(x - x_1)$

m = slope $\qquad$ (x_1, y_1) = point on the line

Distance Traveled

Distance = Rate × Time

Quadratic Formula

For $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagorean Triples

(3, 4, 5)	(5, 12, 13)	(7, 24, 25)
(8, 15, 17)	(9, 40, 41)	(12, 35, 37)

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Sum and Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Circles

Circumference: $2 \times \pi \times r = \pi \times d$

Area: $\pi \times r^2$

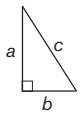
For radius r

Arc Length: $\frac{x}{360} \times 2 \times \pi \times r$

Sector Area: $\frac{x}{360} \times \pi \times r^2$

For central angle of x degrees

Pythagorean Theorem



$a^2 + b^2 = c^2$

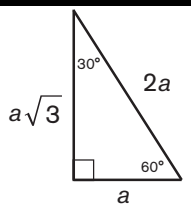
Given A(x_1, y_1) and B(x_2, y_2)

Distance from A to B = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

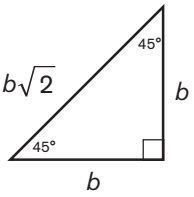
Midpoint of $\overline{AB}$ = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope of $\overline{AB}$ = $\frac{y_2 - y_1}{x_2 - x_1}$

Special Right Triangles



30-60-90 Right Triangle



45-45-90 Right Triangle

Area of Polygons

Square	side length s	s^2
Rectangle	length l, width w	$l \times w$
Parallelogram	base b, height h	$b \times h$
Trapezoid	bases b_1, b_2, height h	$\frac{1}{2}(b_1 + b_2) \times h$
Rhombus	diagonals d_1, d_2	$\frac{1}{2} \times d_1 \times d_2$
Triangle	base b, height h	$\frac{1}{2} \times b \times h$
Triangle	semi-perimeter s, side lengths a, b, c	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length s	$\frac{s^2\sqrt{3}}{4}$

Polygon Angles (n sides)

Sum of the interior angle measures:

$180 \times (n - 2)$

Central angle measure of a regular polygon:

$\frac{360}{n}$

Interior angle measure of a regular polygon:

$\frac{180 \times (n - 2)}{n}$ or $180 - \frac{360}{n}$

Solid	Dimensions	Surface Area	Volume
Cube	side length s	$6 \times s^2$	s^3
Rectangular Prism	length l, width w, height h	$2 \times (l \times w + w \times h + l \times h)$	$l \times w \times h$
Circular Cylinder	base radius r, height h	$2 \times \pi \times r \times h + 2 \times \pi \times r^2$	$\pi \times r^2 \times h$
Circular Cone	base radius r, height h	$\pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$	$\frac{1}{3} \times \pi \times r^2 \times h$
Sphere	radius r	$4 \times \pi \times r^2$	$\frac{4}{3} \times \pi \times r^3$
Pyramid	base area B, height h		$\frac{1}{3} \times B \times h$

Vocabulary & Terms

The following list is representative of terminology used in the problems but **should not** be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
apex
arithmetic mean
arithmetic sequence
base ten
binary
bisect
box-and-whisker plot
center
chord
circumscribe
coefficient
collinear
common divisor
common factor
common fraction
complementary angles
congruent
convex
coordinate plane/system
coplanar
counting numbers
counting principle
diagonal of a polygon
diagonal of a polyhedron
digit-sum
dilation
direct variation
divisor
domain of a function
edge
equiangular
equidistant
expected value
exponent
exterior angle of a polygon
factor
finite
frequency distribution
frustum
function
GCF (GCD)
geometric sequence
hemisphere
image(s) of a point(s) *(under a transformation)*
improper fraction
infinite series
inscribe
integer
interior angle of a polygon
intersection
inverse variation
irrational number
isosceles
lateral edge
lateral surface area
lattice point(s)
LCM
median of a set of data
median of a triangle
mixed number
mode(s) of a set of data
multiplicative inverse *(reciprocal)*
natural number
obtuse angle
ordered pair
origin
palindrome
parallel
Pascal's Triangle
percent increase/decrease
perpendicular
planar
polyhedron
polynomial
prime factorization
principal square root
proper divisor
proper factor
proper fraction
quadrant
quadrilateral
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
reflection
regular polygon
relatively prime
revolution
right angle
right polyhedron
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
slope
space diagonal
square root
stem-and-leaf plot
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
transformation
translation
triangular numbers
trisect
twin primes
union
unit fraction
variable
whole number
y-intercept

ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.
4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.
6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Measurement Stretch

	Answer	Difficulty			
1.	12	(1)	6.	13	(4)
2.	16	(4)	7.	$8\frac{1}{2}$	(3)
3.	3	(4)	8.	42	(3)
4.	44.3	(3)	9.	0.028	(3)
5.	4.2	(3)	10.	5	(4)

Expected Value Stretch

	Answer	Difficulty			
11.	$4\frac{1}{3}$	(2)	16.	20.5	(3)
12.	12.75	(2)	17.	1.2	(4)
13.	3	(3)	18.	2.1	(3)
14.	97/13	(4)	19.	0	(3)
15.	23.3	(4)	20.	100	(3)

Transformations Stretch

	Answer	Difficulty			
21.	5	(2)	26.	(10, 2)	(5)
22.	16	(3)	27.	8	(4)
23.	(7, −4)	(3)	28.	55	(5)
24.	5	(5)	29.	87/4	(6)
25.	(−3, 5)	(3)	30.	$2\sqrt{17}$	(5)

Warm-Up 1

	Answer	Difficulty			
31.	7	(2)	36.	105	(2)
32.	$4\frac{1}{6}$	(2)	37.	26	(3)
33.	1/2	(2)	38.	1*	(2)
34.	20	(2)	39.	50	(3)
35.	1	(3)	40.	580	(3)

Warm-Up 2

	Answer	Difficulty			
41.	120	(2)	46.	36	(3)
42.	36	(2)	47.	50/3	(3)
43.	22	(2)	48.	289/576	(3)
44.	4	(2)	49.	1	(3)
45.	32	(3)	50.	44	(3)

Warm-Up 3

	Answer	Difficulty			
51.	36	(3)	56.	$2\frac{1}{4}$	(4)
52.	2.52	(1)	57.	3	(3)
53.	24	(2)	58.	6	(4)
54.	22	(3)	59.	13	(3)
55.	200	(4)	60.	119	(4)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

Warm-Up 4

	Answer	Difficulty
61.	73	(3)
62.	5	(3)
63.	8:53	(2)
64.	29	(2)
65.	20 or 20.00	(4)
66.	24	(4)
67.	50	(4)
68.	27	(4)
69.	364	(2)
70.	176	(3)

Warm-Up 5

	Answer	Difficulty
71.	69 or 69.00	(4)
72.	392 or 392.00	(3)
73.	52,413	(2)
74.	108	(3)
75.	8	(4)
76.	15 or 15.00	(5)
77.	15/2	(5)
78.	50	(3)
79.	11/29	(3)
80.	9	(5)

Warm-Up 6

	Answer	Difficulty
81.	18	(2)
82.	1/2	(5)
83.	7/15	(4)
84.	17	(4)
85.	1878	(4)
86.	337	(5)
87.	3 or 3.00	(4)
88.	101	(3)
89.	35	(4)
90.	35	(4)

Warm-Up 7

	Answer	Difficulty
91.	15	(4)
92.	25	(5)
93.	3	(3)
94.	78	(3)
95.	2	(3)
96.	-1	(4)
97.	87	(3)
98.	8217902	(2)
99.	28	(5)
100.	12	(4)

Warm-Up 8

	Answer	Difficulty
101.	$6\frac{2}{3}$	(4)
102.	555	(3)
103.	242	(3)
104.	13/32	(4)
105.	12	(4)
106.	16	(3)
107.	$10\sqrt{5}$	(4)
108.	433	(4)
109.	10	(4)
110.	126.5	(5)

Warm-Up 9

	Answer	Difficulty
111.	1740	(3)
112.	3:52	(2)
113.	15	(2)
114.	590	(5)
115.	555	(4)
116.	40	(4)
117.	7	(3)
118.	27	(5)
119.	19/108	(5)
120.	70	(4)

Warm-Up 10

Answer	Difficulty	Answer	Difficulty
121. 6	(3)	126. 1/4	(5)
122. 10	(3)	127. 6	(4)
123. 30	(4)	128. 96	(3)
124. 414	(4)	129. 90	(5)
125. 12.0	(4)	130. 80	(4)

Warm-Up 11

Answer	Difficulty	Answer	Difficulty
131. 11	(3)	136. $2\sqrt{13}$	(5)
132. 300	(4)	137. 13	(4)
133. 18	(5)	138. 22	(4)
134. 11	(2)	139. 28	(5)
135. 40	(3)	140. 5000	(3)

Warm-Up 12

Answer	Difficulty	Answer	Difficulty
141. 252	(3)	146. 3	(5)
142. Tuesday	(3)	147. 26	(5)
143. 144	(3)	148. 18	(5)
144. 24	(4)	149. 48	(6)
145. 4	(4)	150. 13	(4)

Warm-Up 13

Answer	Difficulty	Answer	Difficulty
151. 9	(4)	156. 125/4	(5)
152. 55	(4)	157. 5/8	(5)
153. $12\frac{2}{3}$	(3)	158. 70	(5)
154. $\sqrt{2019}$	(3)	159. 10	(4)
155. 20	(4)	160. −102	(6)

Warm-Up 14

Answer	Difficulty	Answer	Difficulty
161. 60	(4)	166. 55	(3)
162. 50	(7)	167. 8	(2)
163. 121	(5)	168. 15	(5)
164. 1/3	(5)	169. 51/100	(7)
165. 1/4	(3)	170. 12	(4)

Workout 1

Answer	Difficulty	Answer	Difficulty
171. 252	(3)	176. 260	(3)
172. 57	(3)	177. 28.3	(4)
173. 8352	(3)	178. 3.4	(3)
174. 87 or 87.00	(2)	179. 10.87	(3)
175. 90	(3)	180. $48\sqrt{3}$	(4)

Workout 2

Answer	Difficulty	Answer	Difficulty
181. 240	(3)	186. 134	(3)
182. 8	(3)	187. 135	(5)
183. 7	(3)	188. 7	(3)
184. 12	(2)	189. 83	(5)
185. 1091	(3)	190. 32	(4)

Workout 3

Answer	Difficulty	Answer	Difficulty
191. 72	(4)	196. 5/2	(5)
192. 1/18	(5)	197. 18	(4)
193. 60	(3)	198. 17.2	(3)
194. 481	(3)	199. 241.3	(5)
195. 89	(3)	200. 120	(3)

Workout 4

Answer	Difficulty	Answer	Difficulty
201. 4.09	(4)	206. 2166	(4)
202. 2.03	(5)	207. 6.54	(4)
203. 8:48	(2)	208. 203 or 203.00	(4)
204. 1.44	(2)	209. 31	(6)
205. 5	(5)	210. 72	(5)

Workout 5

Answer	Difficulty	Answer	Difficulty
211. 3.5	(5)	216. 78	(4)
212. 2/9	(5)	217. -2	(4)
213. 18,144,000	(4)	218. 33,554,432	(3)
214. 47	(5)	219. 1216.35	(4)
215. 10	(4)	220. $\sqrt{3}/3$	(5)

Workout 6

Answer	Difficulty	Answer	Difficulty
221. 17	(3)	226. 7.8	(6)
222. 145	(5)	227. 1103	(4)
223. 113,400	(5)	228. 225	(4)
224. 2096.40	(4)	229. 56	(4)
225. 4	(3)	230. $3\sqrt{15}$	(5)

Workout 7

Answer	Difficulty	Answer	Difficulty
231. $2\sqrt{3}$	(5)	236. $12\sqrt{15}$	(4)
232. 3205.53	(4)	237. 5.6	(7)
233. 21.6	(6)	238. 0.176	(5)
234. 11	(3)	239. 0.08	(4)
235. 121	(4)	240. $8 + \sqrt{73}$ or $\sqrt{73} + 8$	(5)

Workout 8

Answer	Difficulty	Answer	Difficulty
241. 72	(3)	246. 130	(5)
242. 150.7	(6)	247. 6/5	(5)
243. 2,073,600	(5)	248. 660	(4)
244. 364	(5)	249. 7	(5)
245. 3	(5)	250. $6\sqrt{6}$	(7)

MATHCOUNTS Problems Mapped to Common Core State Standards (CCSS)

Forty-two states, the District of Columbia, four territories and the Department of Defense Education Activity (DoDEA) have voluntarily adopted the Common Core State Standards (CCSS). As such, MATHCOUNTS considers it beneficial for teachers to see the connections between the *2018-2019 MATHCOUNTS School Handbook* problems and the CCSS. MATHCOUNTS not only has identified a general topic and assigned a difficulty level for each problem but also has provided a CCSS code in the Problem Index (pages 54-55). A complete list of the Common Core State Standards can be found at www.corestandards.org.

The CCSS for mathematics cover K-8 and high school courses. MATHCOUNTS problems are written to align with the NCTM Standards for Grades 6-8. As one would expect, there is great overlap between the two sets of standards. MATHCOUNTS also recognizes that in many school districts, algebra and geometry are taught in middle school, so some MATHCOUNTS problems also require skills taught in those courses.

In referring to the CCSS, the Problem Index code for each of the Standards for Mathematical Content for grades K-8 begins with the grade level. For the Standards for Mathematical Content for high school courses (such as algebra or geometry), each code begins with a letter to indicate the course name. The second part of each code indicates the domain within the grade level or course. Finally, the number of the individual standard within that domain follows. Here are two examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts utilized in MATHCOUNTS problems are not specifically mentioned in the CCSS. Two examples are the Fundamental Counting Principle (FCP) and special right triangles. In cases like these, if a related standard could be identified, a code for that standard was used. For example, problems using the FCP were coded 7.SP.8, S-CP.8 or S-CP.9 depending on the context of the problem; SP → Statistics and Probability (the domain), S → Statistics and Probability (the course) and CP → Conditional Probability and the Rules of Probability. Problems based on special right triangles were given the code G-SRT.5 or G-SRT.6, explained above.

There are some MATHCOUNTS problems that either are based on math concepts outside the scope of the CCSS or based on concepts in the standards for grades K-5 but are obviously more difficult than a grade K-5 problem. When appropriate, these problems were given the code SMP for Standards for Mathematical Practice. The CCSS include the Standards for Mathematical Practice along with the Standards for Mathematical Content. The SMPs are (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure and (8) Look for and express regularity in repeated reasoning.

PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 49. For an explanation of the CCSS codes refer to page 53.

Measurement

1	(1)	6.RP.3
2	(4)	4.MD.2
3	(4)	6.RP.3
4	(3)	4.MD.2
5	(3)	SMP
6	(4)	6.RP.3
7	(3)	4.MD.2
8	(3)	4.MD.2
9	(3)	4.MD.2
10	(4)	6.RP.3
42	(2)	6.G.1
61	(3)	7.G.5
81	(2)	SMP
97	(3)	6.RP.3
102	(3)	4.MD.2
112	(2)	4.OA.2
248	(4)	6.G.1

Solid Geometry

36	(2)	6.G.2
116	(4)	7.G.6
153	(3)	7.G.6
182	(3)	G-GMD.3
191	(4)	7.G.6
192	(5)	G-GMD.3
199	(5)	G-GMD.4
202	(5)	G-GMD.3
208	(4)	7.G.6
212	(5)	G-GMD.3
242	(6)	8.G.9

Coordinate Geometry

Transformations Stretch[1]		
46	(3)	6.G.1
100	(4)	6.G.1
123	(4)	7.G.6
129	(5)	SMP
148	(5)	8.G.1
158	(5)	6.G.3

[1] CCSS 8.G.3

Plane Geometry

38	(2)	4.G.3
56	(4)	G-C.2
58	(4)	8.G.8
60	(4)	4.G.2
68	(4)	G-SRT.5
70	(3)	G-CO.10
77	(5)	7.RP.2
78	(3)	4.MD.3
80	(5)	7.G.5
82	(5)	G-C.2
84	(4)	SMP
90	(4)	7.G.5
110	(5)	G-C.2
120	(4)	7.G.5
125	(4)	8.G.8
126	(5)	G-SRT.5
133	(5)	7.G.5
136	(5)	G-SRT.6
144	(4)	8.G.7
146	(5)	S-CP.9
150	(4)	7.G.6
154	(3)	8.G.7
166	(3)	SMP
180	(4)	7.G.6
181	(3)	6.G.1
187	(5)	8.G.5
189	(5)	7.G.6
193	(3)	SMP
196	(5)	8.G.8
211	(5)	7.G.6
220	(5)	G-SRT.6
230	(5)	7.G.6
233	(6)	7.G.1
236	(4)	8.G.7
240	(5)	8.G.7
250	(7)	G-C.2

Algebraic Expressions & Equations

34	(2)	3.NBT.2
35	(3)	8.EE.8
41	(2)	6.EE.3
45	(3)	8.EE.8
49	(3)	6.EE.9
55	(4)	A-SSE.2
62	(3)	8.EE.7
67	(4)	7.EE.4
76	(5)	8.EE.8
86	(5)	8.EE.8
87	(4)	8.EE.8
94	(3)	6.EE.7
96	(4)	A-SSE.2
109	(4)	6.EE.3
111	(3)	6.EE.9
121	(3)	6.EE.3
122	(3)	8.EE.8
130	(4)	6.RP.3
142	(3)	6.RP.3
147	(5)	A-REI.4
151	(4)	A-APR.5
161	(4)	6.EE.7
176	(3)	6.RP.3
186	(3)	6.EE.2
201	(4)	7.RP.3
217	(4)	A-CED.1
231	(5)	A-REI.4
237	(7)	A-SSE.3
245	(5)	A-REI.2
246	(5)	8.EE.8

General Math

52	(1)	7.NS.2
63	(2)	SMP
69	(2)	3.NBT.2
174	(2)	6.NS.3
203	(2)	4.MD.2

Sequences, Series & Patterns

93	(3)	SMP
132	(4)	F-BF.2
139	(5)	F-BF.2
156	(5)	F-BF.2
160	(6)	A-APR.5
195	(3)	F-IF.3
222	(5)	8.G.7
235	(4)	F-BF.2
249	(5)	F-BF.2

Problem Solving (Misc.)

57	(3)	SMP
91	(4)	6.EE.7
92	(5)	SMP
162	(7)	8.EE.8
218	(3)	SMP
232	(4)	SMP
234	(3)	SMP
244	(5)	SMP

Probability, Counting & Combinatorics

Expected Value Stretch[2]		
33	(2)	7.SP.7
53	(2)	S-CP.9
83	(4)	7.SP.8
99	(5)	S-CP.9
103	(3)	S-CP.9
104	(4)	7.SP.8
106	(3)	S-CP.9
119	(5)	7.SP.8
138	(4)	S-CP.9
141	(3)	7.G.6
152	(4)	SMP
157	(5)	7.SP.8
164	(5)	S-CP.9
169	(7)	7.SP.8
170	(4)	S-CP.9
210	(5)	S-CP.9
213	(4)	S-CP.9
214	(5)	7.SP.8
223	(5)	S-CP.9
238	(5)	7.SP.8

Statistics

48	(3)	6.SP.2
66	(4)	6.SP.5
75	(4)	6.SP.2
85	(4)	6.SP.2
95	(3)	6.SP.5
108	(4)	6.SP.2
115	(4)	SMP
118	(5)	6.SP.2
128	(3)	6.SP.5
140	(3)	7.SP.6
149	(6)	SMP
159	(4)	6.SP.5
168	(5)	6.SP.5
175	(3)	6.SP.2
179	(3)	6.SP.2
204	(2)	6.SP.4
216	(4)	6.SP.5
226	(6)	G-GMD.4

Proportional Reasoning

43	(2)	6.RP.3
54	(3)	6.RP.3
71	(4)	6.RP.3
89	(4)	6.RP.3
135	(3)	6.RP.3
155	(4)	6.RP.3
165	(3)	G-SRT.5
172	(3)	7.RP.3
177	(4)	6.RP.1
178	(3)	6.RP.3
185	(3)	7.RP.3
190	(4)	7.RP.3
194	(3)	7.RP.3
206	(4)	6.RP.3
207	(4)	6.RP.3
215	(4)	7.RP.3
224	(4)	6.RP.3
227	(4)	6.RP.3
229	(4)	7.NS.3
239	(4)	7.NS.3

Logic

31	(2)	S-CP.9
40	(3)	SMP
72	(3)	SMP
124	(4)	SMP
137	(4)	SMP
163	(5)	SMP
171	(3)	7.NS.3
200	(3)	SMP
221	(3)	7.SP.5

Number Theory

32	(2)	3.NBT.2
37	(3)	6.SP.2
39	(3)	5.NBT.2
44	(2)	8.EE.2
59	(3)	8.EE.8
64	(2)	4.OA,3
73	(2)	SMP
74	(3)	8.EE.2
79	(3)	6.NS.1
88	(3)	4.OA.4
98	(2)	SMP
105	(4)	6.RP.3
107	(4)	6.SP.2
113	(2)	6.RP.3
114	(5)	F-BF.2
117	(3)	6.EE.7
127	(4)	N-RN.2
131	(3)	6.NS.4
134	(2)	4.OA.4
145	(4)	6.NS.4
167	(2)	7.NS.2
173	(3)	SMP
183	(3)	6.NS.4
184	(2)	SMP
188	(3)	S-CP.9
197	(4)	SMP
225	(3)	SMP
228	(4)	4.OA.4
243	(5)	SMP

Percents & Fractions

47	(3)	7.NS.3
50	(3)	7.RP.3
51	(3)	7.NS.3
65	(4)	7.NS.3
101	(4)	6.EE.7
143	(3)	8.EE.7
198	(3)	7.RP.3
205	(5)	7.EE.4
209	(6)	7.NS.3
219	(4)	7.RP.3
241	(3)	7.RP.3
247	(5)	6.RP.3

[2] CCSS S-MD.2

SOLUTIONS

The solutions provided here are only *possible* solutions. It is very likely that you or your students will come up with additional—and perhaps more elegant—solutions. Happy solving!

SOLUTIONS ARE AVAILABLE EXCLUSIVELY FOR REGISTERED COACHES AND THOSE WHO PURCHASE A HARD COPY OF THE HANDBOOK.

- Learn more about how you can register your school for the Competition Series at www.mathcounts.org/competition.
- Purchase a copy of the *2018-2019 MATHCOUNTS School Handbook* at www.mathcounts.org/store.

Other MATHCOUNTS Programs

MATHCOUNTS was founded in 1983 as a way to provide new avenues of engagement in math for middle school students. MATHCOUNTS began solely as a competition, but has grown to include 3 unique but complementary programs: the **MATHCOUNTS Competition Series**, the **National Math Club** and the **Math Video Challenge**. Your school can participate in all 3 MATHCOUNTS programs!

THE NATIONAL MATHCLUB powered by MATHCOUNTS®

The **National Math Club** is a free enrichment program that provides teachers and club leaders with resources to run a math club. The materials provided through the National Math Club are designed to engage students of all ability levels—not just the top students—and are a great supplement for classroom teaching. This program emphasizes collaboration and provides students with an enjoyable, pressure-free atmosphere in which they can learn math at their own pace.

Active clubs also can earn rewards by having a minimum number of club members participate (based on school/organization/group size). **There is no cost to sign up for the National Math Club**, and registration is open to schools and non-school groups that consist of at least 4 U.S. students in grades 6-8 that have regular in-person meetings. More information can be found at **www.mathcounts.org/club**, and the 2018-2019 School Registration Form is included on the next page.

MATHVIDEO CHALLENGE produced by MATHCOUNTS®

The **Math Video Challenge** is an innovative program that challenges students to work in teams of 4 to create a video explaining the solution to a MATHCOUNTS handbook problem and demonstrating its real-world application. This project-based activity builds math, communication and collaboration skills.

Students post their videos to the contest website, where the general public votes for the best videos. The 100 videos with the most votes advance to judging rounds, in which 20 semifinalists and, later, 4 finalists are selected. This year's finalists will present their videos to the students competing at the 2019 Raytheon MATHCOUNTS National Competition, and the 224 Mathletes will vote to determine the winner. Members of the winning team receive college scholarships. **Registration is completely free** and open to all students in grades 6-8. More information can be found at **videochallenge.mathcounts.org**.

The fastest way to register is online at **www.mathcounts.org/clubreg**!

THE NATIONAL MATHCLUB® powered by MATHCOUNTS®

2018-2019 **SCHOOL** REGISTRATION FORM

This registration form is for U.S. middle schools only. To register a non-school group (such as a Girl Scout troop, Boys and Girls Club Chapter or math circle) for the National Math Club, please go to www.mathcounts.org/club to review eligibility requirements and register.

*indicates required information

STEP 1 Tell Us About Your School

☒ **U.S. School (Including Homeschools) with Students in Grades 6-8**
One school can have multiple clubs, as long as each club has a different club leader.

School Name* _____

School Type (check one)* ☐ Public ☐ Charter ☐ Private ☐ Homeschool ☐ Virtual

Title I School? (check one)* ☐ Yes ☐ No

Overseas U.S. schools must provide additional information below:

My school is sponsored by the U.S. Department of: ☐ *Defense (DoDDS)* ☐ *State*

Country _____

Estimated Number of Students Participating in Club (Minimum 4)* _____

Typically, what percentage of your club members are female?* _____%

☐ **My school participated in the National Math Club last year (2017-2018).**

How did you hear about the National Math Club?* ☐ Prior Participation ☐ Word-of-mouth ☐ Email ☐ Mailing (postcard or newsletter) ☐ Internet Search ☐ Workshop/Conference

STEP 2 Tell Us About You

Club Leader First & Last Name* _____

Club Leader Email* _____

Club Leader Alternate Email* _____

Your email and alternate email address will not be made public or shared and will only be used by MATHCOUNTS.

Club Street Address* _____

City, State and ZIP Code* _____

☐ **I participated in MATHCOUNTS when I was in middle school. → Please tell us more info. below!**

When? (for example, 1999-02) _____ Where? (state/territory) _____

Which Program(s)? ☐ MATHCOUNTS Competition Series → Highest Level Reached: _____
☐ The National Math Club (formerly MATHCOUNTS Club Program)
☐ Math Video Challenge (formerly Reel Math Challenge)

STEP 3 Turn in Your Form

IMPORTANT! *By submitting this form you attest your group consists of at least 4 U.S. students in grades 6-8 who meet in person regularly, and is therefore eligible to participate in the National Math Club. The club leader will receive an emailed confirmation once this registration has been processed.*

Mail or email a scanned copy of this completed form to:

Address: MATHCOUNTS Registration | 1420 King Street | Alexandria, VA 22314 *Email:* reg@mathcounts.org

MATHCOUNTS® COMPETITION SERIES

The fastest way to register is online at **www.mathcounts.org/compreg**!

2018–2019 SCHOOL REGISTRATION FORM

STEP 1: SCHOOL & COACH INFORMATION

*required information

Coach First and Last Name* _____ Is this your first year coaching?* ☐ Yes ■ No

Coach Email* _____

Coach Phone _____ Coach Alternate Email _____

School Name* _____ School Type* ☐ Public ☐ Charter ☐ Private ☐ Homeschool ☐ Virtual

School Street Address* _____ City, State & ZIP* _____

County/District* _____ MATHCOUNTS Chapter (if known) _____

Ship Materials to:* ☐ School Address ☐ Alternate Address Here: _____

Total # of students participating in school's MATHCOUNTS meetings* _____ ☐ My school previously participated in the MATHCOUNTS Competition Series.

Co-Coach Name _____ Co-Coach Email _____

I participated in MATHCOUNTS when I was in middle school. → When? (for ex. 1999-02) _____ Where? (state/territory) _____

Which Program(s)? ☐ Competition Series → Highest Level Reached: _____ ☐ National Math Club ☐ Math Video Challenge (formerly Reel Math Challenge)

My co-coach participated in MATHCOUNTS when s/he was in middle school. → When? (for ex. 1999-02) _____ Where? (state/territory)_____

Which Program(s)? ☐ Competition Series → Highest Level Reached: _____ ☐ National Math Club ☐ Math Video Challenge (formerly Reel Math Challenge)

Overseas U.S. schools must be affiliated with the U.S. Dept. of Defense or State and must provide additional information below.

My school is sponsored by the U.S. Dept. of: ☐ Defense (DoDDS). ☐ State. Country _____

STEP 2: # OF COMPETITORS

Please circle below the *number of students* you will bring to the Chapter Competition and the *associated cost* (depending on the date your registration is postmarked).

# of Students to Be Registered (Ind = Individual; T = Team)	**1** (1 Ind)	**2** (2 Ind)	**3** (3 Ind)	**4** (1 T)	**5** (1 T + 1 Ind)	**6** (1 T + 2 Ind)	**7** (1 T + 3 Ind)	**8** (1 T + 4 Ind)	**9** (1 T + 5 Ind)	**10** (1 T + 6 Ind)
*Early Bird Rate: $30 per student (postmarked **by** Nov. 2, 2018)*	**$30**	**$60**	**$90**	**$120**	**$150**	**$180**	**$210**	**$240**	**$270**	**$300**
*Regular Rate: $35 per student (postmarked **by** Dec. 14, 2018)*	**$35**	**$70**	**$105**	**$140**	**$175**	**$210**	**$245**	**$280**	**$315**	**$350**
*Late Registration: +$20 late fee (postmarked **after** Dec. 14, 2018)*	**$55**	**$90**	**$125**	**$160**	**$195**	**$230**	**$265**	**$300**	**$335**	**$370**

☐ My school qualifies for the **50% Title I discount**, so the Amount Due in Step 3 will be half the amount circled above. Principal signature required to verify Title I eligibility.

Principal Name _____ Principal Signature _____

STEP 3: PAYMENT

Amount Due $_____ ☐ Credit Card ☐ Check *(payable to MATHCOUNTS Foundation)* ☐ Money Order ☐ Purchase Order #_____ *(must include copy of P.O.)*

Do NOT include any credit card information on this form. *Within 5 business days you will receive an email invoice enabling you to pay by credit card.*

IMPORTANT! *By submitting this form you (1) agree to adhere to the rules of the MATHCOUNTS Competition Series; (2) attest you have the school administration's permission to register students for this program under this school's name; and (3) affirm the above named school is a U.S. school eligible for this program and* not *an academic or enrichment center. The coach will receive an emailed confirmation and receipt once this registration has been processed.*

Mail or email a scanned copy of this completed form to:

Address: MATHCOUNTS Registration | 1420 King Street | Alexandria, VA 22314 *Email:* reg@mathcounts.org

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